Higher Mathematics

2024 Paper 2

Time allowed = 1 hr 30 mins Marks available = 65

For each question, you can click below to view the worked solutions for each question. You can also view this paper's marking scheme below;

www.sqa.org.uk/pastpapers/papers/instructions/2024/mi_NH_Mathematics_Paper-2_2024.pdf

Remember to record your percentage for this paper in your analysis grid (your score ÷ 65 × 100).

FORMULAE LIST

Circle

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar product

$\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

Table of standard derivatives

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

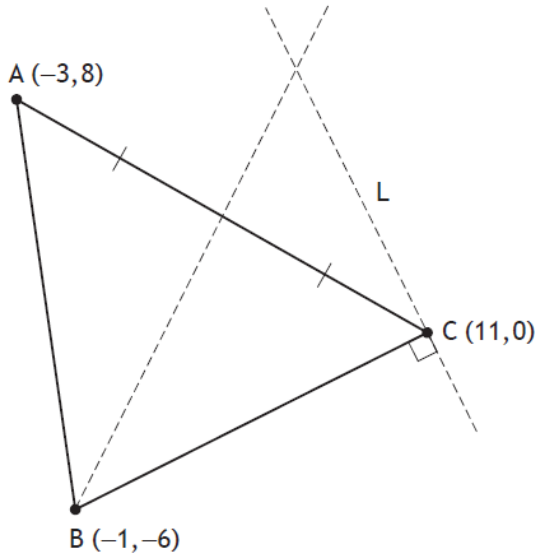
Table of standard integrals

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

Total marks — 65

Attempt ALL questions

1. Triangle ABC has vertices A(−3, 8), B(−1, −6) and C(11, 0).



(a) Find the equation of the median through B. 3

(b) Find the equation of L, the line perpendicular to BC passing through C. 3

(c) Determine the coordinates of the point of intersection of the median through B and the line L. 2

Click here to view the video solutions.

Video Lessons: 1·8 Bronze Outcome 1, 1·6 Silver Outcome 2 and 1·9 Silver Outcome 2

2. A curve has equation $y = \frac{8}{x^3}$, $x > 0$.

   Find the equation of the tangent to this curve at the point where $x = 2$. 5

Click here to view the video solutions.

Video Lesson: 6·3 Silver Outcome 2

3. The coordinates of points D, E and F are given by D(2, −3, 4), E(1, 1, −2) and F(3, 2, 1).

   (a) Express $\overline{ED}$ and $\overline{EF}$ in component form. 2

   (b) (i) Calculate $\overline{ED}.\overline{EF}$. 1

   (ii) Hence, or otherwise, calculate the size of angle DEF. 4

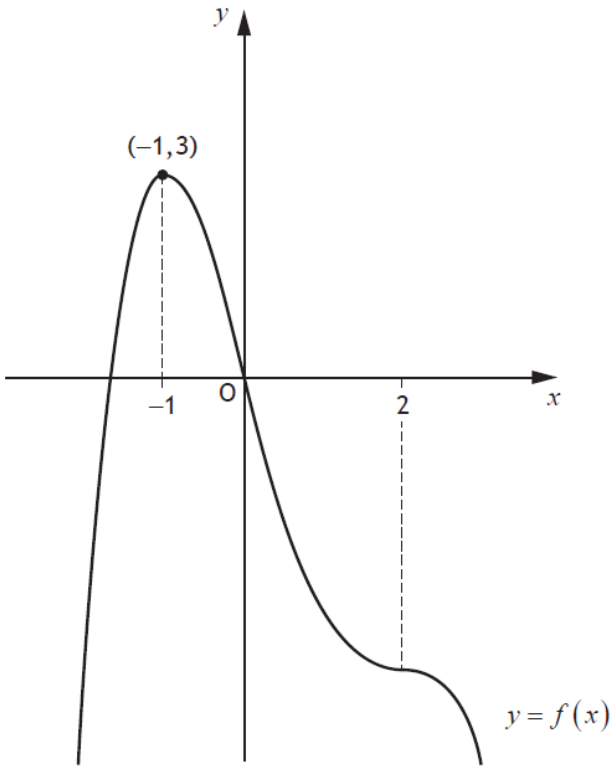
Click here to view the video solutions.

Video Lesson: 12·5 Outcome 1

4. The diagram shows the graph of a quartic function $y = f(x)$.

A maximum turning point occurs at $(-1, 3)$.

The graph of $y = f(x)$ also has a point of inflection at $x = 2$.



(a) Determine the coordinates of the maximum turning point on the graph of $y = f(x-4)+2$. 2

(b) On the diagram in your answer booklet, sketch the graph of $y = f'(x)$. 3

Click here to view the video solutions.

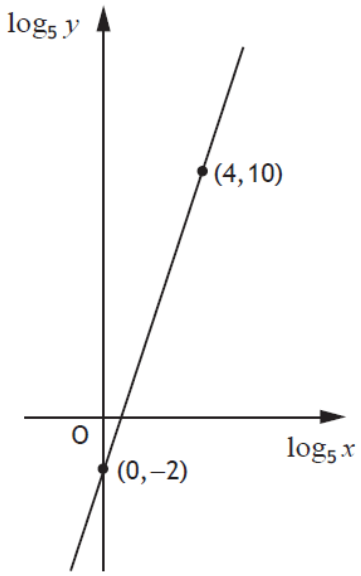
Video Lessons: 4·1 Gold Outcome 3, 6·4 Gold Outcome 3

5. Evaluate $\int_0^{\frac{\pi}{7}} \sin 5x \, dx$. 3

Click here to view the video solutions.

Video Lesson: 13·2 Silver Outcome 2

6. Two variables, x and y, are connected by the equation $y = ax^b$.
The graph of $\log_5 y$ against $\log_5 x$ is a straight line as shown.

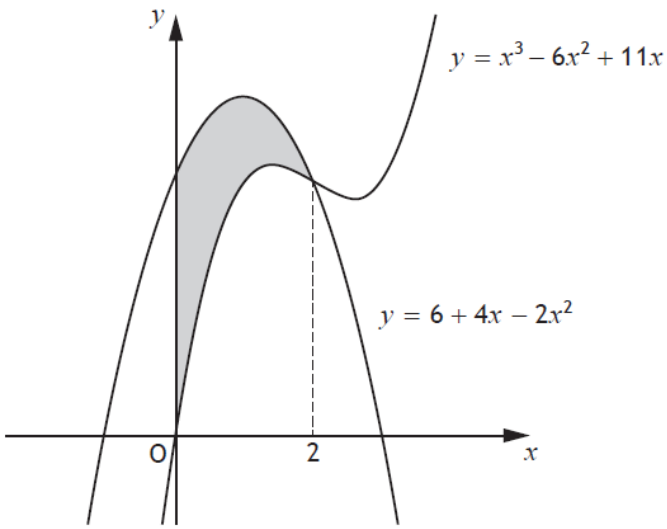


Find the values of a and b. 5

Click here to view the video solutions.

Video Lessons: 14·4 Gold Outcome 3

7. The diagram shows the curve with equation $y = x^3 - 6x^2 + 11x$ intersecting the curve with equation $y = 6 + 4x - 2x^2$ at $x = 2$.



Calculate the shaded area. 5

Click here to view the video solutions.

Video Lesson: 9·4 Gold Outcome 3

8. Functions f and g are defined on $\mathbb{R}$, the set of real numbers, by:

- $f(x) = 2x^2 - 18$
- $g(x) = x + 1$.

(a) Find an expression for $f(g(x))$. 2

(b) Find the values of x for which $\dfrac{1}{f(g(x))}$ is undefined. 2

Click here to view the video solutions.

Video Lessons: 3·2 Silver Outcome 2, 3·1 Bronze Outcome 1

9. (a) Determine the coordinates of the stationary points on the curve with equation $y = \frac{1}{3}x^3 - x^2 - 3x + 1$. 4

(b) Hence, determine the greatest and least values of y in the interval $-1 \leq x \leq 6$. 2

Click here to view the video solutions.

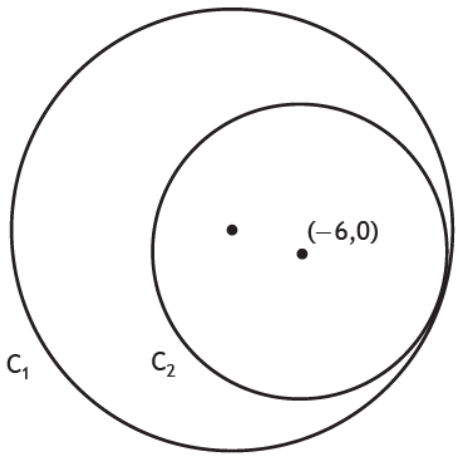
Video Lesson: 6·6 Outcome 1

10. The circle C_1 has equation $x^2 + y^2 + 18x - 2y - 8 = 0$.

(a) Find the centre and radius of C_1. 2

A second circle, C_2, touches C_1 internally.

The centre of C_2 is $(-6, 0)$.



(b) Determine the equation of C_2. 2

Click here to view the video solutions.

Video Lessons: 11·1 Bronze Outcome 1 and Gold Outcome 3

11. The number of electric vehicles worldwide can be modelled by

$$N = 6.8e^{kt}$$

where:

- N is the estimated number of vehicles in millions
- t is the number of years since the end of 2020
- k is a constant.

(a) Use the model to estimate the number of electric vehicles worldwide at the end of 2020. **1**

At the end of 2030, it is estimated there will be 125 million electric vehicles worldwide.

(b) Determine the value of k. **4**

Click here to view the video solutions.

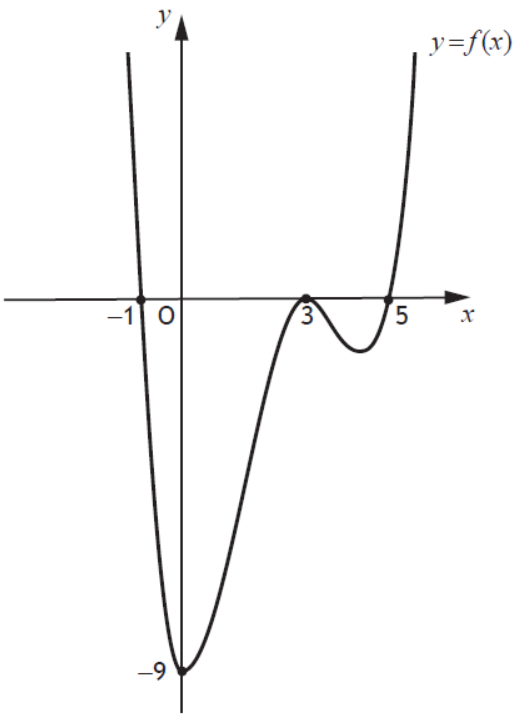
Video Lesson: 14·3 Gold Outcome 3

12. Solve the equation $2\sin 2x° - \sin^2 x° = 0,\ 0 \leq x < 360.$ **5**

Click here to view the video solutions.

Video Lesson: 10·2 Gold Outcome 3

13. The diagram shows the graph of $y = f(x)$, where $f(x)$ is a quartic function.



Express $f(x)$ in the form $f(x) = k(x+a)^2(x+b)(x+c)$. 3

Click here to view the video solutions.

Video Lesson: 8·1 Gold Outcome 3

[END OF QUESTION PAPER]