Higher Mathematics

2024 Paper 1

Time allowed = 1 hr 15 mins Marks available = 55

For each question, you can click below to view the worked solutions for each question. You can also view this paper's marking scheme below;

https://www.sqa.org.uk/pastpapers/papers/instructions/2024/mi_NH_Mathematics_Paper-1-Non-calculator_2024.pdf

Remember to record your percentage for this paper in your analysis grid (your score ÷ 55 × 100).

FORMULAE LIST

Circle

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar product

$\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

Table of standard derivatives

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

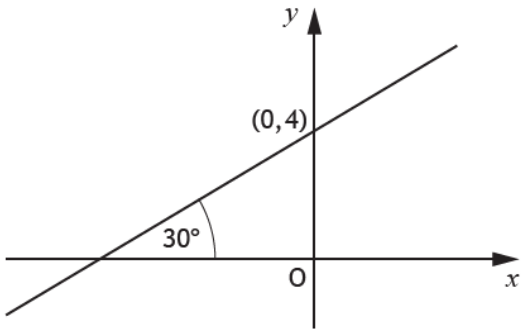
Table of standard integrals

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

Total marks — 55

Attempt ALL questions

1. A line passes through the point $(0, 4)$ and makes an angle of 30° with the positive direction of the x-axis as shown in the diagram.



Determine the equation of the line. 3

Click here to view the worked solutions.

Video Lesson: 1·3 Gold Outcome 3

2. A sequence is defined by the recurrence relation $u_{n+1} = \frac{1}{5}u_n + 12$ with $u_1 = 20$.

(a) Calculate the value of u_2. 1

(b) (i) Explain why this sequence approaches a limit as $n \to \infty$. 1

(ii) Calculate this limit. 2

Click here to view the worked solutions.

Video Lessons: 2·1 Silver Outcome 2 and 2·2 Silver Outcome 2

3. Given that $y = \left(5x^2 + 3\right)^7$, find $\frac{dy}{dx}$. 2

Click here to view the worked solutions.

Video Lesson: 13·1 Bronze Outcome 1

4. P and Q have coordinates $(-6, 1, 2)$ and $(-1, 11, -8)$ respectively.
Find the coordinates of the point R which divides PQ in the ratio 2:3. 2

Click here to view the worked solutions.

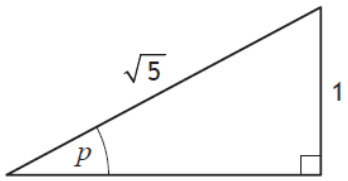
Video Lesson: 12·3 Silver Outcome 2

5. A function, h, is defined by $h(x) = 2x^3 - 7$ where $x \in \mathbb{R}$.
Find the inverse function, $h^{-1}(x)$. 3

Click here to view the worked solutions.

Video Lesson: 3·3 Outcome 1

6. The right-angled triangle in the diagram is such that $\sin p = \frac{1}{\sqrt{5}}$ and $0 < p < \frac{\pi}{4}$.



(a) Determine the value of:

(i) $\sin 2p$ 3

(ii) $\cos 2p$. 1

(b) Hence determine the value of $\sin 4p$. 1

Click here to view the worked solutions.

Video Lessons: 10·1 Bronze Outcome 1 and Silver Outcome 2

7. The line $y = 2x$ is a tangent to the circle with equation $x^2 + y^2 - 14x - 8y + 45 = 0$.
Determine the coordinates of the point of contact. 4

Click here to view the worked solutions.

Video Lesson: 11·3 Silver Outcome 2

8. The equation $x^2+(m-4)x+(2m-3)=0$ has no real roots.
Determine the range of values for m. 4
Justify your answer.

Click here to view the worked solutions.

Video Lesson: 8·54 Gold Outcome 3

9. Express $\log_a 5+\log_a 80-2\log_a 10$ in the form $\log_a k$ where k is a positive integer. 3

Click here to view the worked solutions.

Video Lesson: 14·1 Gold Outcome 3

10. (a) Show that $(x-1)$ is a factor of $2x^4+3x^3-4x^2-3x+2$. 2

(b) Hence, or otherwise, factorise $2x^4+3x^3-4x^2-3x+2$ fully. 4

Click here to view the worked solutions.

Video Lesson: 7·1 Gold Outcome 3

11. (a) Express $\cos x° + \sqrt{3}\sin x°$ in the form $k\cos(x-a)°$, where $k > 0$ and $0 < a < 360$. 4

(b) Hence, or otherwise, sketch the graph with equation $y = \cos x° + \sqrt{3}\sin x°$, $0 \leq x \leq 360$.

Use the diagram provided in your answer booklet. 3

Click here to view the worked solutions.

Video Lessons: 15·1 Bronze Outcome 1 and 15·2 Gold Outcome 3

12. The function f is given by $f(x) = 12\sqrt[3]{x}$, $x > 0$.

When $x = a$ the rate of change of f with respect to x is 1.

Determine the value of a. 4

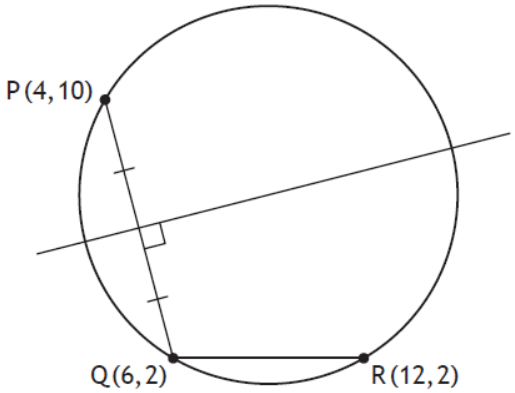
Click here to view the worked solutions.

Video Lessons: 6·2 Silver Outcome 2

13. P and Q are the points (4, 10) and (6, 2) respectively.

(a) Find the equation of the perpendicular bisector of PQ. 4

The point R has coordinates (12, 2).
A circle passes through the points P, Q and R.
The chord QR is horizontal.



(b) Find the equation of the circle. 4

Click here to view the worked solutions.

Video Lessons: 1·8 Gold Outcome 3, 11·1 Bronze Outcome 1

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