Higher Mathematics

2023 Paper 2

Time allowed = 1 hr 30 mins Marks available = 65

For each question, you can click below to view the worked solutions for each question. You can also view this paper's marking scheme below;

www.sqa.org.uk/pastpapers/papers/instructions/2023/mi_NH_Mathematics_Paper-2_2023.pdf

Remember to record your percentage for this paper in your analysis grid (your score ÷ 65 × 100).

FORMULAE LIST

Circle

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar product

$\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

Table of standard derivatives

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

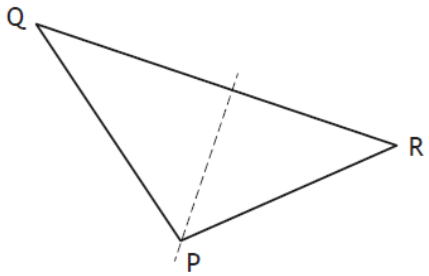
Table of standard integrals

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

Total marks — 65

Attempt ALL questions

1. Triangle PQR has vertices P(5, −1), Q(−2, 8) and R(13, 3).



(a) Find the equation of the altitude from P. 3

(b) Calculate the angle that the side PR makes with the positive direction of the x-axis. 2

Click here to view the video solutions.

Video Lessons: 1·8 Silver Outcome 2, 1·3 Gold Outcome 3

2. Find the equation of the tangent to the curve with equation $y = 2x^5 - 3x$ at the point where $x = 1$. 4

Click here to view the video solutions.

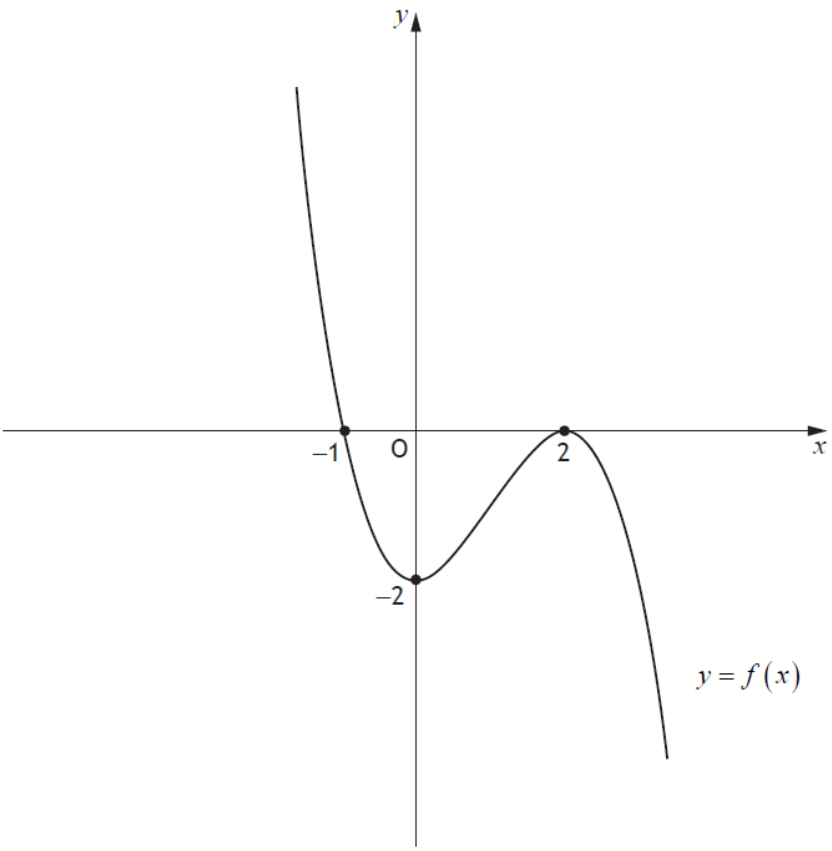
Video Lesson: 6·3 Silver Outcome 2

3. Find $\int 7\cos\left(4x+\frac{\pi}{3}\right)dx$. 2

Click here to view the video solutions.

Video Lesson: 13·2 Silver Outcome 2

4. The diagram shows the cubic graph of $y=f(x)$, with stationary points at (2, 0) and (0, −2).



On the diagram in your answer booklet, sketch the graph of $y=2f(-x)$. 2

Click here to view the video solutions.

Video Lesson: 4·1 Gold Outcome 3

5. A function, f, is defined by $f(x) = (3-2x)^4$, where $x \in \mathbb{R}$.
Calculate the rate of change of f when $x = 4$. 3

Click here to view the video solutions.

Video Lesson: 13·1 Bronze Outcome 1

6. A function $f(x)$ is defined by $f(x) = \frac{2}{x} + 3$, $x > 0$.
Find the inverse function, $f^{-1}(x)$. 3

Click here to view the video solutions.

Video Lessons: 3·3 Outcome 1

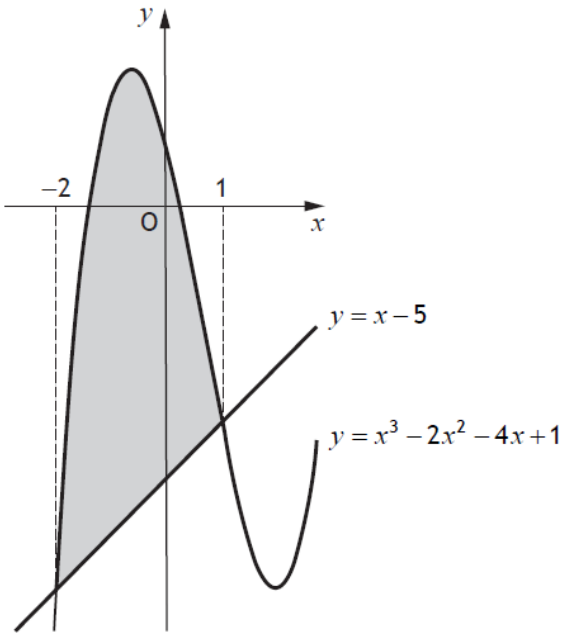
7. Solve the equation $\sin x° + 2 = 3\cos 2x°$ for $0 \leq x < 360$. 5

Click here to view the video solutions.

Video Lesson: 10·2 Gold Outcome 3

8. The diagram shows part of the curve with equation $y = x^3 - 2x^2 - 4x + 1$ and the line with equation $y = x - 5$.

The curve and the line intersect at the points where $x = -2$ and $x = 1$.



Calculate the shaded area. 5

Click here to view the video solutions.

Video Lesson: 9·4 Silver Outcome 2

9. (a) Express $7\cos x° - 3\sin x°$ in the form $k\sin(x + a)°$ where $k > 0$, $0 < a < 360$. 4

(b) Hence, or otherwise, find:

(i) the maximum value of $14\cos x° - 6\sin x°$ 1

(ii) the value of x for which it occurs where $0 \leq x < 360$. 2

Click here to view the video solutions.

Video Lessons: 15·1 Silver Outcome 2, 15·2 Silver Outcome 2

10. Determine the range of values of x for which the function $f(x) = 2x^3 + 9x^2 - 24x + 6$ is strictly decreasing. 4

Click here to view the video solutions.

Video Lesson: 6·4 Bronze Outcome 1

11. Circle C_1 has equation $(x-4)^2 + (y+2)^2 = 37$.
Circle C_2 has equation $x^2 + y^2 + 2x - 6y - 7 = 0$.

(a) Calculate the distance between the centres of C_1 and C_2. 3

(b) Hence, show that C_1 and C_2 intersect at two distinct points. 3

Click here to view the video solutions.

Video Lesson: 11·4 Bronze Outcome 1

12. A curve, for which $\frac{dy}{dx} = 8x^3 + 3$, passes through the point $(-1, 3)$.
Express y in terms of x. 4

Click here to view the video solutions.

Video Lesson: 9·3 Outcome 1

13. A patient is given a dose of medicine.

The concentration of the medicine in the patient's blood is modelled by

$$C_t = 11e^{-0.0053\,t}$$

where:

- t is the time, in minutes, since the dose of medicine was given
- C_t is the concentration of the medicine, in mg/l, at time t.

(a) Calculate the concentration of the medicine 30 minutes after the dose was given. 1

The dose of medicine becomes ineffective when its concentration falls to 0.66 mg/l.

(b) Calculate the time taken for this dose of the medicine to become ineffective. 3

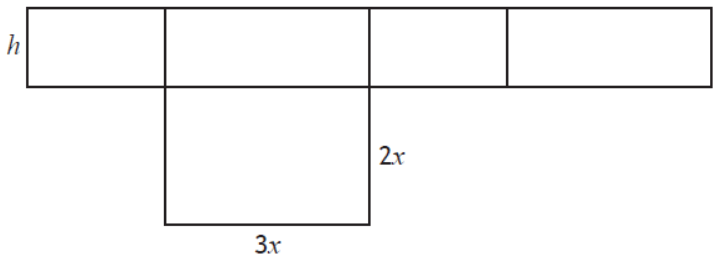
Click here to view the video solutions.

Video Lesson: 14·3 Gold Outcome 3

14. A net of an open box is shown.

The box is a cuboid with height h centimetres.

The base is a rectangle measuring $3x$ centimetres by $2x$ centimetres.



(a) (i) Express the area of the net, A cm^2, in terms of h and x. 1

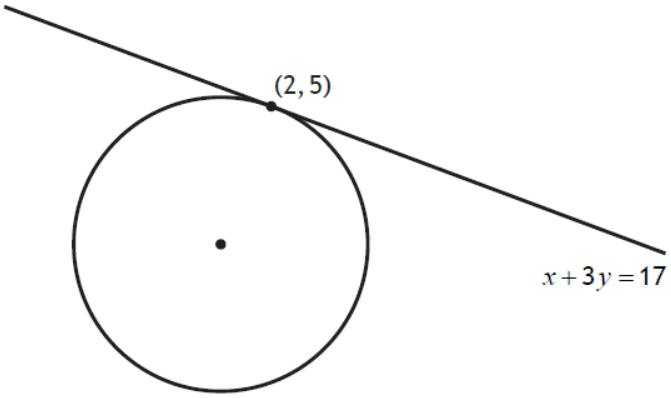
(ii) Given that $A = 7200$ cm^2, show that the volume of the box, V cm^3, is given by $V = 4320x - \frac{18}{5}x^3$. 2

(b) Determine the value of x that maximises the volume of the box. 4

Click here to view the video solutions.

Video Lesson: 6·7 Outcome 1

15. The line $x+3y=17$ is a tangent to a circle at the point $(2, 5)$.



The centre of the circle lies on the y-axis.

Find the coordinates of the centre of the circle. 4

Click here to view the video solutions.

Video Lesson: 11·2 Gold Outcome 3

[END OF QUESTION PAPER]