Higher Mathematics

2022 Paper 1

Time allowed = 1 hr 15 mins   Marks available = 55

For each question, you can click below to view the worked solutions for each question. You can also view this paper's marking scheme below;

www.sqa.org.uk/pastpapers/papers/instructions/2022/mi_NH_Mathematics_Paper-1-Non-calculator_2022.pdf

Remember to record your percentage for this paper in your analysis grid (your score ÷ 55 × 100).

FORMULAE LIST

Circle

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar product $\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$= 2\cos^2 A - 1$

$= 1 - 2\sin^2 A$

Table of standard derivatives

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\dfrac{1}{a}\cos ax + c$
$\cos ax$	$\dfrac{1}{a}\sin ax + c$

Total marks — 55
Attempt ALL questions

1. Determine the equation of the line perpendicular to $5x + 2y = 7$, passing through $(-1, 6)$.   **3**

Click here to view the worked solutions.

Video Lesson: 1·6 Silver Outcome 2

2. Evaluate $2\log_3 6 - \log_3 4$.   **3**

Click here to view the worked solutions.

Video Lesson: 14·1 Gold Outcome 3

3. A function, h, is defined by $h(x) = 4 + \frac{1}{3}x$, where $x \in \mathbb{R}$.
   Find the inverse function, $h^{-1}(x)$.   **3**

Click here to view the worked solutions.

Video Lesson: 3·3 Outcome 1
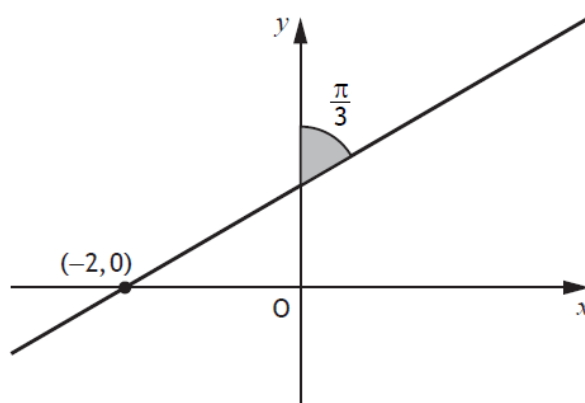
4. Differentiate $y = \sqrt{x^3} - 2x^{-1}$, where $x > 0$.   3

Click here to view the worked solutions.

Video Lesson: 6·1 Silver Outcome 2

5. A line makes an angle of $\frac{\pi}{3}$ radians with the y-axis, and passes through the point $(-2, 0)$ as shown below.

    Determine the equation of the line.   3

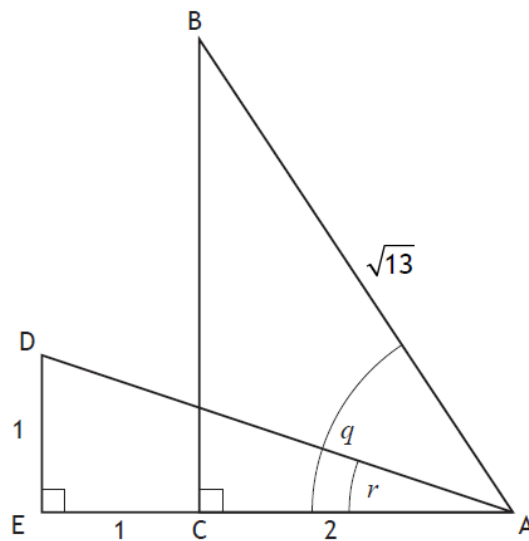Click here to view the worked solutions.

Video Lesson: 1·3 Gold Outcome 3

6. Evaluate $\int_{-5}^{2} (10 - 3x)^{-\frac{1}{2}} \, dx$.   4

Click here to view the worked solutions.

Video Lesson: 13·2 Bronze Outcome 1

7. Triangles ABC and ADE are both right angled.

   Angle BAC = q and angle DAE = r as shown in the diagram.

   (a) Determine the value of:

       (i) $\sin r$ 1

       (ii) $\sin q$.   1

   (b) Hence determine the value of $\sin(q-r)$.   3

Click here to view the worked solutions.

Video Lesson: 10·1 Gold Outcome 3

8. Solve $\log_6 x + \log_6 (x+5) = 2$, where $x>0$.   4

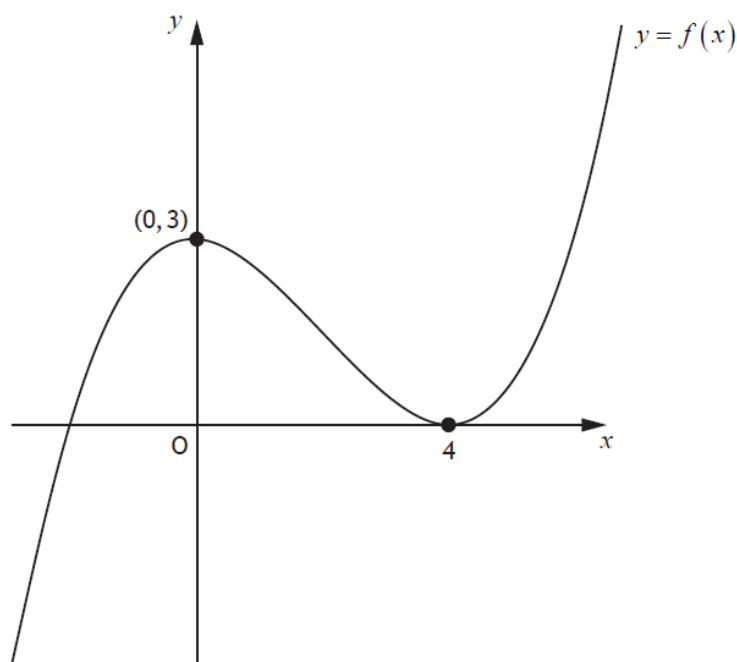Click here to view the worked solutions.

Video Lesson: 14·2 Gold Outcome 3

9. Solve the equation $\cos 2x° = 5\cos x° - 3$ for $0 \le x < 360$.   5

Click here to view the worked solutions.

Video Lesson: 10·2 Gold Outcome 3

10. The diagram shows the graph of a cubic function with equation $y = f(x)$.
     The curve has stationary points at $(0, 3)$ and $(4, 0)$.

(a) Sketch the graph of $y = 2f(x) + 1$.   3

     Use the diagram provided in the answer booklet.

(b) State the coordinates of the stationary points on the graph of $y = f\left(\dfrac{1}{2}x\right)$.   1

Click here to view the worked solutions.

Video Lesson: 4·1 Gold Outcome 3

11. Express $2x^2 + 12x + 23$ in the form $p(x+q)^2 + r$.   **3**

Click here to view the worked solutions.

Video Lessons: 8·2 Bronze Outcome 1

12. Given that $f(x) = 4\sin\left(3x - \dfrac{\pi}{3}\right)$, evaluate $f'\left(\dfrac{\pi}{6}\right)$.   **3**
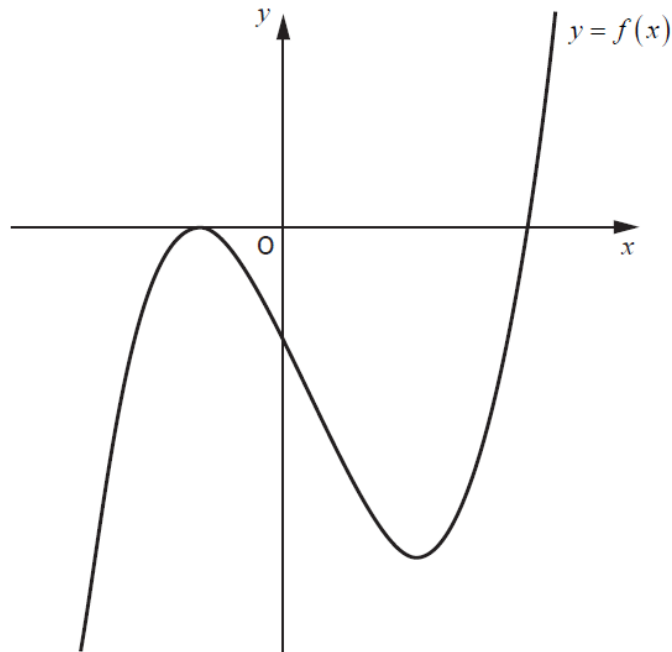
Click here to view the worked solutions.

Video Lessons: 13·1 Silver Outcome 2

13. (a) (i) Show that $(x+2)$ is a factor of $f(x) = x^3 - 2x^2 - 20x - 24$.   2

 (ii) Hence, or otherwise, solve $f(x) = 0$.   3

The diagram shows the graph of $y = f(x)$.

(b) The graph of $y = f(x-k)$, $k > 0$ has a stationary point at $(1, 0)$.

State the value of k.   1

Click here to view the worked solutions.

Video Lesson: 7·1 Bronze Outcome 1

14. C_1 is the circle with equation $(x-7)^2+(y+5)^2=100$.

   (a)   (i) State the centre and radius of C_1.   2

         (ii) Hence, or otherwise, show that the point P$(-2,7)$ lies outside C_1.   2

   C_2 is a circle with centre P and radius r.

   (b) Determine the value(s) of r for which circles C_1 and C_2 have exactly one point of
        intersection.   2

Click here to view the worked solutions.

Video Lessons: 11·4 Silver Outcome 2

[END OF QUESTION PAPER]