Higher Mathematics

2022 Paper 2

Time allowed = 1 hr 30 mins Marks available = 65

For each question, you can click below to view the worked solutions for each question. You can also view this paper's marking scheme below;

www.sqa.org.uk/pastpapers/papers/instructions/2022/mi_NH_Mathematics_Paper-2_2022.pdf

Remember to record your percentage for this paper in your analysis grid (your score ÷ 65 × 100).

FORMULAE LIST

Circle

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar product

$\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

Table of standard derivatives

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

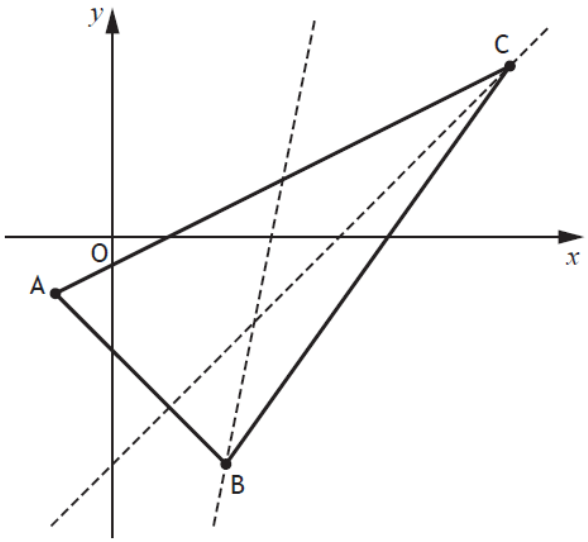
Table of standard integrals

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

Total marks — 65

Attempt ALL questions

1. Triangle ABC has vertices A(−1, −1), B(2, −4) and C(7, 3).



(a) Find the equation of the altitude through C. 3

(b) Find the equation of the median through B. 3

(c) Determine the coordinates of the point of intersection of the altitude through C and the median through B. 2

Click here to view the video solutions.

Video Lessons: 1·8 Silver Outcome 2, 1·8 Bronze Outcome 1, 1·9 Silver Outcome 2

2. The equation $2x^2 - 8x + (4 - p) = 0$ has two real and distinct roots.

Determine the range of values for p. 3

Click here to view the video solutions.

Video Lesson: 8·4 Gold Outcome 3

3. (a) Express $4\sin x + 5\cos x$ in the form $k\sin(x+a)$ where $k > 0$ and $0 < a < 2\pi$. 4

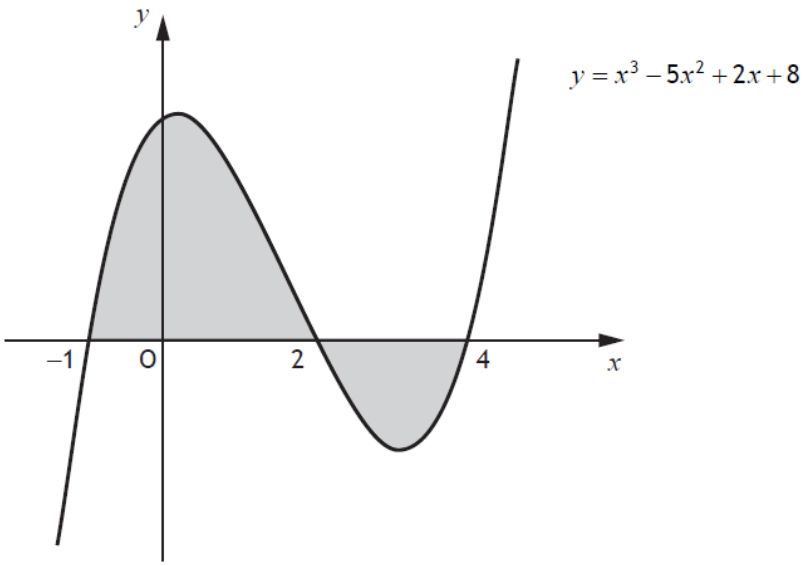
(b) Hence solve $4\sin x + 5\cos x = 5.5$ for $0 \leq x < 2\pi$. 3

Click here to view the video solutions.

Video Lesson: 15·1 Bronze Outcome 1, 15·2 Bronze Outcome 1

4. The graph shown has equation $y = x^3 - 5x^2 + 2x + 8$.

The total shaded area is bounded by the curve and the x-axis.



(a) Calculate the shaded area above the x-axis. 4

(b) Hence calculate the total shaded area. 3

Click here to view the video solutions.

Video Lesson: 9·4 Bronze Outcome 1

5. Functions f and g are given by $f(x) = x^2 - 2$ and $g(x) = 3x + 5$, $x \in \mathbb{R}$.

   (a) Find expressions for:

   (i) $f(g(x))$ and 2

   (ii) $g(f(x))$. 1

   (b) Determine the range of values of x for which $f(g(x)) < g(f(x))$. 4

Click here to view the video solutions.

Video Lessons: 3·2 Silver Outcome 2, 8·3 Silver Outcome 2

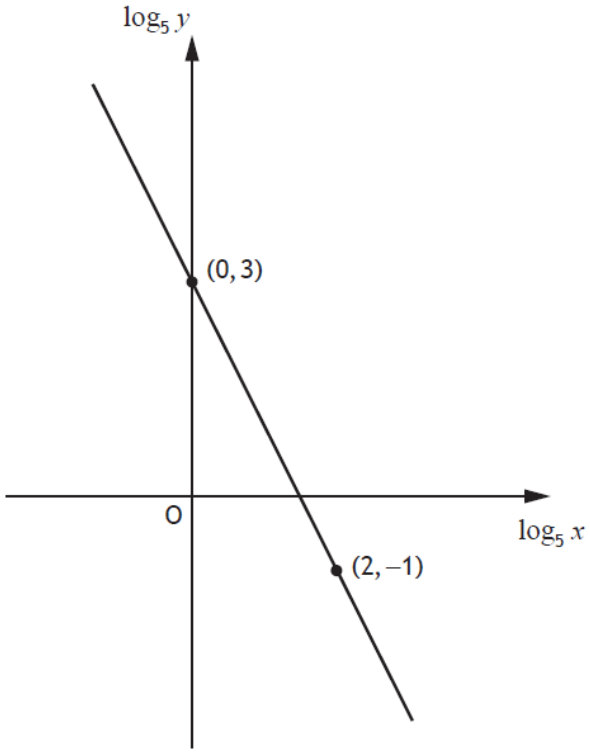
6. A curve with equation $y = f(x)$ is such that $\frac{dy}{dx} = 1 - \frac{3}{x^2}$, where $x > 0$.
   The curve passes through the point $(3, 6)$.

   Express y in terms of x. 5

Click here to view the video solutions.

Video Lessons: 9·3 Outcome 1

7. Two variables, x and y, are connected by the equation $y = kx^n$.
The graph of $\log_5 y$ against $\log_5 x$ is a straight line as shown.



Find the values of k and n. 5

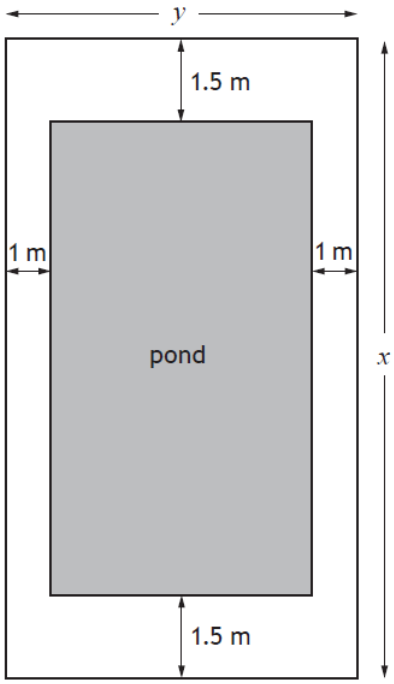
Click here to view the video solutions.

Video Lesson: 14·4 Gold Outcome 3

8. A rectangular plot consists of a rectangular pond surrounded by a path.

The length and breadth of the plot are x metres and y metres respectively.

The path is 1.5 metres wide at the ends of the pond and 1 metre wide along the other sides as shown.



The total area of the **pond and path together** is 150 square metres.

(a) Show that the area of the pond, A square metres, is given by

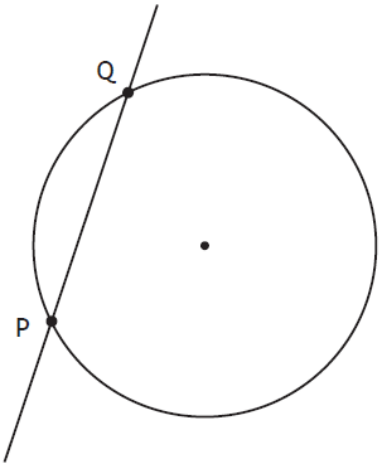
$$A(x) = 156 - 2x - \frac{450}{x}.$$ **3**

(b) Determine the maximum area of the pond. **6**

Click here to view the video solutions.

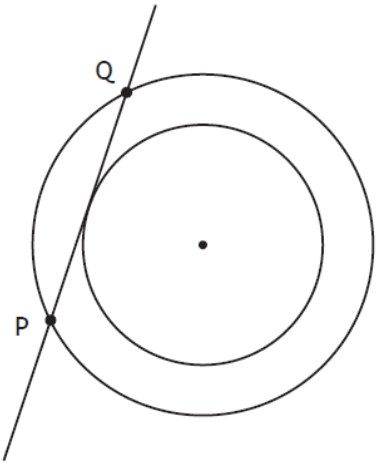
Video Lesson: 6·7 Outcome 3

9. The line $y = 3x + 7$ intersects the circle $x^2 + y^2 - 4x - 6y - 7 = 0$ at the points P and Q.



(a) Find the coordinates of P and Q. 5

PQ is a tangent to a second, smaller circle.
This circle is concentric with the first.



(b) Determine the equation of the smaller circle. 4

Click here to view the video solutions.

Video Lesson: 11·3 Bronze Outcome 1

10. The heptathlon is an athletics contest made up of seven events.

Athletes score points for each event.

In the 200 metres event, the points are calculated using the formula

$$P = 4.99087(42.5 - T)^{1.81}$$

where P is the number of points awarded, and T is the athlete's time, in seconds.

(a) Calculate how many points would be awarded for a time of 24.55 seconds in the 200 metres event. 1

In the long jump event, the points are calculated using the formula

$$P = 0.188807(D - 210)^{k}$$

where P is the number of points awarded, D is the distance jumped, in centimetres, and k is a constant.

(b) Given that 850 points are awarded for a jump of 600 cm, calculate the value of k. 4

Click here to view the video solutions.

Video Lesson: 14·3 Gold Outcome 3

[END OF QUESTION PAPER]