Higher Mathematics

2023 Paper 1

Time allowed = 1 hr 15 mins Marks available = 55

For each question, you can click below to view the worked solutions for each question. You can also view this paper's marking scheme below;

www.sqa.org.uk/pastpapers/papers/instructions/2023/mi_NH_Mathematics_Paper-1-Non-Calculator_2023.pdf

Remember to record your percentage for this paper in your analysis grid (your score ÷ 55 × 100).

FORMULAE LIST

Circle

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar product

$\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

Table of standard derivatives

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

Total marks — 55

Attempt ALL questions

1. Given that $y = x^{\frac{5}{3}} - \frac{10}{x^4}$, where $x \neq 0$, find $\frac{dy}{dx}$. 3

Click here to view the worked solutions.

Video Lesson: 6·1 Silver Outcome 2

2. P and Q are the points $(-2, 6)$ and $(10, 0)$.
   Find the equation of the perpendicular bisector of PQ. 4

Click here to view the worked solutions.

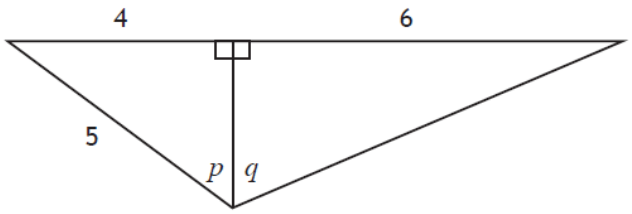
Video Lesson: 1·8 Gold Outcome 3

3. Solve $\log_5 x - \log_5 3 = 2$. 3

Click here to view the worked solutions.

Video Lesson: 14·2 Silver Outcome 2

4. The diagram shows two right-angled triangles with angles p and q as marked.



(a) Determine the value of:

(i) $\cos p$ 1

(ii) $\cos q$. 1

(b) Hence determine the value of $\cos(p+q)$. 3

Click here to view the worked solutions.

Video Lesson: 10·1 Gold Outcome 3

5. The equation $2x^2+(3p-2)x+p=0$ has equal roots.

Determine the possible values of p. 3

Click here to view the worked solutions.

Video Lesson: 8·4 Gold Outcome 3

6. Find $\int\left(2x^5-6\sqrt{x}\right)dx,\ x\geq 0$. 4

Click here to view the worked solutions.

Video Lesson: 9·1 Silver Outcome 2

7. (a) Evaluate $\log_2 5 + \log_2 \frac{1}{40}$. 2

(b) Given that $a \in \mathbb{R}$ and that $\log_8 a$ is negative, state the range of possible values of a. 1

Click here to view the worked solutions.

Video Lesson: 14·1 Silver Outcome 2

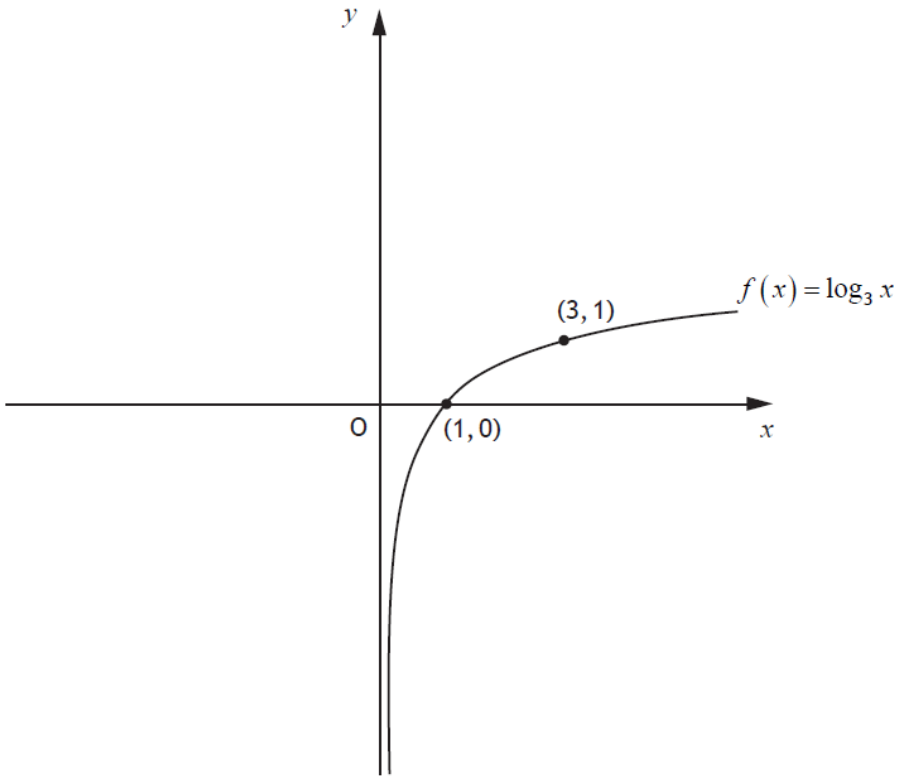
8. A function, f, is defined on $\mathbb{R}$, the set of real numbers, by $f(x) = x^3 + 3x^2 - 9x + 5$.

Find the coordinates of the stationary points of f and determine their nature. 6

Click here to view the worked solutions.

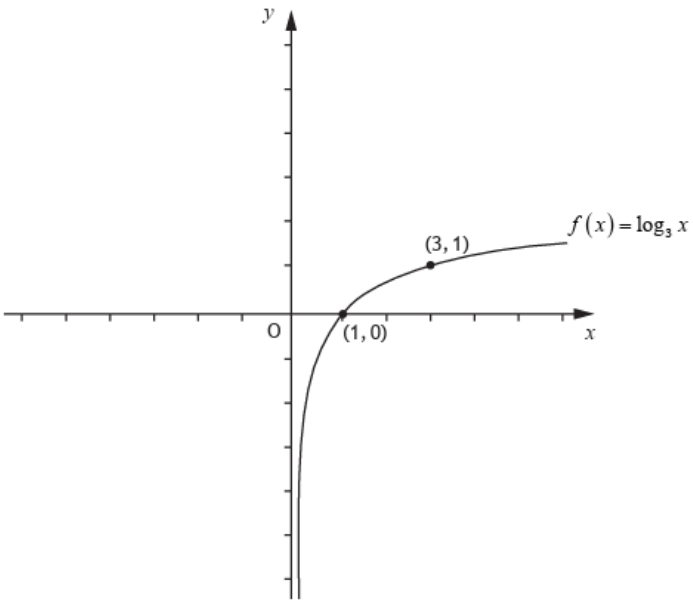
Video Lessons: 6·5 Bronze Outcome 1, Silver Outcome 2

9. The diagram shows the graph of the function $f(x) = \log_3 x$, where $x > 0$.



The inverse function, f^{-1}, exists.

On the diagram in your answer booklet, sketch the graph of $y = f^{-1}(x) - 1$. 3



Click here to view the worked solutions.

Video Lesson: 4·2 Gold Outcome 3

10. (a) Show that $(x+5)$ is a factor of $x^4+3x^3-7x^2+9x-30$. 2

(b) Hence, or otherwise, solve $x^4+3x^3-7x^2+9x-30=0$, $x\in\mathbb{R}$. 5

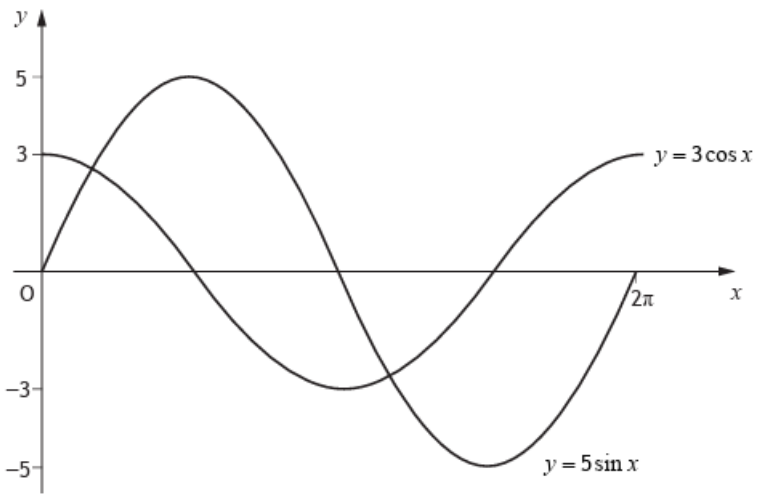
Click here to view the worked solutions.

Video Lesson: 7·1 Gold Outcome 3

11. (a) Evaluate $\int_{\frac{\pi}{2}}^{\pi}(5\sin x-3\cos x)\,dx$. 3

The diagram in your answer booklet shows the graphs with equations $y=5\sin x$ and $y=3\cos x$, $0\le x\le 2\pi$.

(b) On the diagram in your answer booklet, shade the area represented by the integral in (a). 1



Click here to view the worked solutions.

Video Lessons: 13·2 Gold Outcome 2

12. Express $-2x^2-12x+7$ in the form $a(x+b)^2+c$. 3

Click here to view the worked solutions.

Video Lessons: 8·2 Silver Outcome 2

13. Functions f and g are defined by:

- $f(x)=2\sin x$, where $0<x<\frac{\pi}{2}$
- $g(x)=2x$, where $0<x<\frac{\pi}{4}$

(a) (i) Evaluate $f\left(g\left(\frac{\pi}{6}\right)\right)$. 1

(ii) Determine an expression for $f(g(x))$. 2

(b) (i) Given that $f(p)=\frac{1}{3}$, determine the exact value of $\sin p$. 1

(ii) Hence, determine the exact value of $f(g(p))$. 3

Click here to view the worked solutions.

Video Lessons: 3·2 Silver Outcome 2, 10·1 Bronze Outcome 1

[END OF QUESTION PAPER]