SQA Past paper questions

2017 – Paper 1 – Question 9

A sequence is generated by the recurrence relation $u_{n+1} = m u_n + 6$ where m is a constant.

(a) Given $u_1 = 28$ and $u_2 = 13$, find the value of m. 2

(b) (i) Explain why this sequence approaches a limit as $n \to \infty$. 1

(ii) Calculate this limit. 2

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2016 – Paper 1 – Question 3

A sequence is defined by the recurrence relation $u_{n+1} = \frac{1}{3}u_n + 10$ with $u_3 = 6$.

(a) Find the value of u_4. 1

(b) Explain why this sequence approaches a limit as $n \to \infty$. 1

(c) Calculate this limit. 2

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2015 – Paper 2 – Question 3

A version of the following problem first appeared in print in the 16th Century.

A frog and a toad fall to the bottom of a well that is 50 feet deep.
Each day, the frog climbs 32 feet and then rests overnight. During the night, it slides down $\frac{2}{3}$ of its height above the floor of the well.
The toad climbs 13 feet each day before resting.
Overnight, it slides down $\frac{1}{4}$ of its height above the floor of the well.

Their progress can be modelled by the recurrence relations:

- $f_{n+1} = \frac{1}{3}f_n + 32,$ $f_1 = 32$
- $t_{n+1} = \frac{3}{4}t_n + 13,$ $t_1 = 13$

where f_n and t_n are the heights reached by the frog and the toad at the end of the nth day after falling in.

(a) Calculate t_2, the height of the toad at the end of the second day. 1

(b) Determine whether or not either of them will eventually escape from the well. 5

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2012 – Paper 2 – Question 6

For $0 < x < \frac{\pi}{2}$, sequences can be generated using the recurrence relation

$$u_{n+1} = (\sin x)u_n + \cos 2x, \text{ with } u_0 = 1.$$

(*a*) Why do these sequences have a limit? **2**

(*b*) The limit of one sequence generated by this recurrence relation is $\frac{1}{2}\sin x$.

Find the value(s) of x. **7**

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2007 – Paper 1 – Question 7

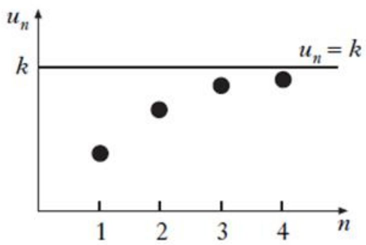
A sequence is defined by the recurrence relation

$$u_{n+1} = \frac{1}{4}u_n + 16, \; u_0 = 0.$$

(*a*) Calculate the values of u_1, u_2 and u_3. **3**

Four terms of this sequence, u_1, u_2, u_3 and u_4 are plotted as shown in the graph.

As $n \to \infty$, the points on the graph approach the line $u_n = k$, where k is the limit of this sequence.



(*b*) (i) Give a reason why this sequence has a limit.

(ii) Find the exact value of k. **3**

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2006 – Paper 1 – Question 4

A sequence is defined by the recurrence relation $u_{n+1} = 0{\cdot}8u_n + 12$, $u_0 = 4$.

(*a*) State why this sequence has a limit. **1**

(*b*) Find this limit. **2**

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2005 – Paper 1 – Question 6

(*a*) The terms of a sequence satisfy $u_{n+1} = ku_n + 5$. Find the value of k which produces a sequence with a limit of 4. **2**

(*b*) A sequence satisfies the recurrence relation $u_{n+1} = mu_n + 5$, $u_0 = 3$.

(i) Express u_1 and u_2 in terms of m.

(ii) Given that $u_2 = 7$, find the value of m which produces a sequence with no limit. **5**

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2004 – Paper 2 – Question 4

A sequence is defined by the recurrence relation $u_{n+1} = ku_n + 3$.

(*a*) Write down the condition on k for this sequence to have a limit. **1**

(*b*) The sequence tends to a limit of 5 as $n \to \infty$. Determine the value of k. **3**

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2000 – Paper 1 – Question 5

Two sequences are generated by the recurrence relations $u_{n+1} = au_n + 10$ and $v_{n+1} = a^2 v_n + 16$.

The two sequences approach the same limit as $n \to \infty$.

Determine the value of a and evaluate the limit. 5

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1999 – Paper 1 – Question 18

Two sequences are defined by the recurrence relations

$$u_{n+1} = 0 \cdot 2u_n + p, \quad u_0 = 1 \quad \text{and}$$
$$v_{n+1} = 0 \cdot 6v_n + q, \quad v_0 = 1.$$

If both sequences have the same limit, express p in terms of q. **(3)**

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