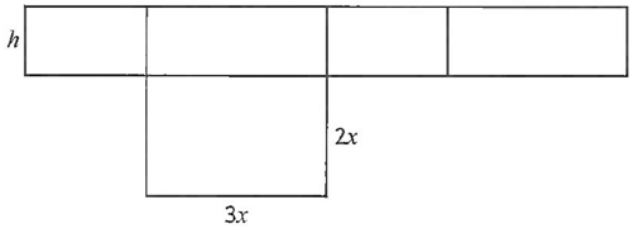
SQA Past paper questions

2023 – Paper 2 – Question 14

A net of an open box is shown.

The box is a cuboid with height h centimetres.

The base is a rectangle measuring $3x$ centimetres by $2x$ centimetres.



(a) (i) Express the area of the net, A cm^2, in terms of h and x. 1

(ii) Given that $A = 7200$ cm^2, show that the volume of the box, V cm^3, is given by $V = 4320x - \frac{18}{5}x^3$. 2

(b) Determine the value of x that maximises the volume of the box. 4

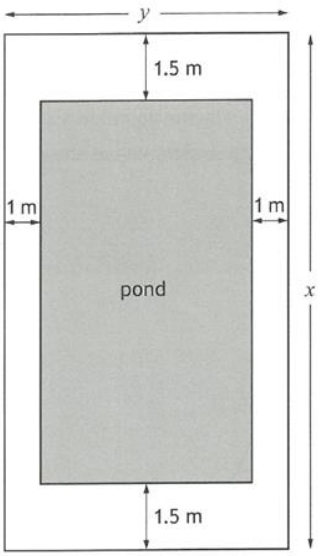
Click here for video solution.

2022 – Paper 2 – Question 8

A rectangular plot consists of a rectangular pond surrounded by a path.

The length and breadth of the plot are x metres and y metres respectively.

The path is 1.5 metres wide at the ends of the pond and 1 metre wide along the other sides as shown.



The total area of the **pond and path together** is 150 square metres.

(a) Show that the area of the pond, A square metres, is given by

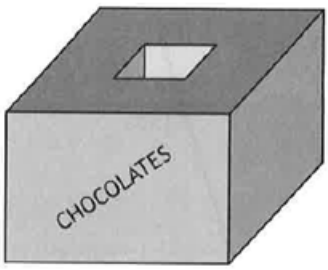
$$A(x) = 156 - 2x - \frac{450}{x}.$$ 3

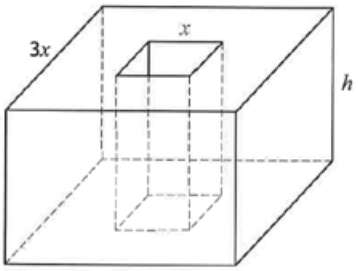
(b) Determine the maximum area of the pond. 6

Click here for video solution.

2019 – Paper 2 – Question 11

A manufacturer of chocolates is launching a new product in novelty shaped cardboard boxes.





The box is a cuboid with a cuboid shaped tunnel through it.

- The height of the box is h centimetres
- The top of the box is a square of side $3x$ centimetres
- The end of the tunnel is a square of side x centimetres
- The volume of the box is $2000\,\text{cm}^3$

(a) Show that the total surface area, A cm^2, of the box is given by

$$A = 16x^2 + \frac{4000}{x}.$$

3

(b) To minimise the cost of production, the surface area, A, of the box should be as small as possible.

Find the minimum value of A. **6**

Click here for video solution.

2018 – Paper 2 – Question 9

A sector with a particular fixed area has radius x cm.

The perimeter, P cm, of the sector is given by

$$P = 2x + \frac{128}{x}.$$

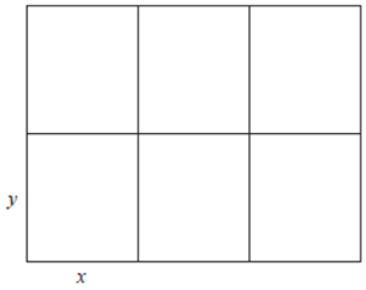
Find the minimum value of P. **6**

Click here for video solution.

2016 – Paper 2 – Question 7

A council is setting aside an area of land to create six fenced plots where local residents can grow their own food.

Each plot will be a rectangle measuring x metres by y metres as shown in the diagram.



(a) The area of land being set aside is 108 m^2.

Show that the total length of fencing, L metres, is given by

$$L(x) = 9x + \frac{144}{x}.$$ 3

(b) Find the value of x that minimises the length of fencing required. 6

Click here for video solution.

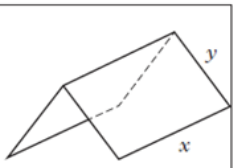
2013 – Paper 2 – Question 7

A manufacturer is asked to design an open-ended shelter, as shown, subject to the following conditions.

Condition 1

The frame of a shelter is to be made of rods of two different lengths:

- x metres for top and bottom edges;
- y metres for each sloping edge.



Condition 2

The frame is to be covered by a rectangular sheet of material.

The total area of the sheet is 24 m^2.

(*a*) Show that the total length, L metres, of the rods used in a shelter is given by

$$L = 3x + \frac{48}{x}.$$ 3

(*b*) These rods cost £8·25 per metre.

To minimise production costs, the total length of rods used for a frame should be as small as possible.

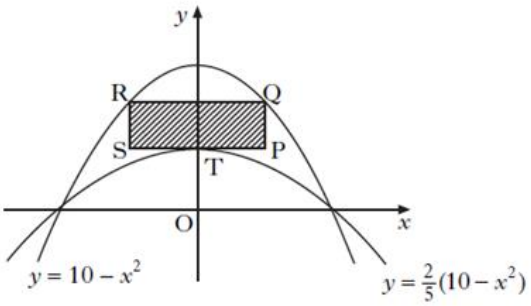
(i) Find the value of x for which L is a minimum.

(ii) Calculate the minimum cost of a frame. 7

Click here for video solution.

2010 – Paper 2 – Question 5

The parabolas with equations $y = 10 - x^2$ and $y = \frac{2}{5}(10 - x^2)$ are shown in the diagram below.



A rectangle PQRS is placed between the two parabolas as shown, so that:

- Q and R lie on the upper parabola;
- RQ and SP are parallel to the x-axis;
- T, the turning point of the lower parabola, lies on SP.

(*a*) (i) If TP = x units, find an expression for the length of PQ.

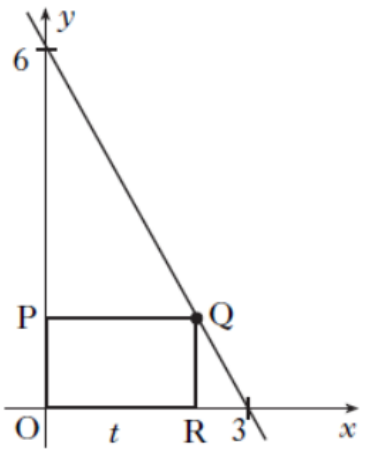
(ii) Hence show that the area, A, of rectangle PQRS is given by 3

$$A(x) = 12x - 2x^3.$$

(*b*) Find the maximum area of this rectangle. 6

Click here for video solution.

2008 – Paper 2 – Question 6



In the diagram, Q lies on the line joining (0, 6) and (3, 0).

OPQR is a rectangle, where P and R lie on the axes and OR = t.

(*a*) Show that QR = $6 - 2t$. 3

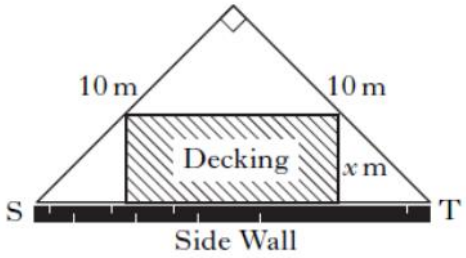
(*b*) Find the coordinates of Q for which the rectangle has a maximum area. 6

Click here for video solution.

2007 – Paper 2 – Question 6

A householder has a garden in the shape of a right-angled isosceles triangle.

It is intended to put down a section of rectangular wooden decking at the side of the house, as shown in the diagram.



(*a*) (i) Find the exact value of ST.

(ii) Given that the breadth of the decking is x metres, show that the area of the decking, A square metres, is given by

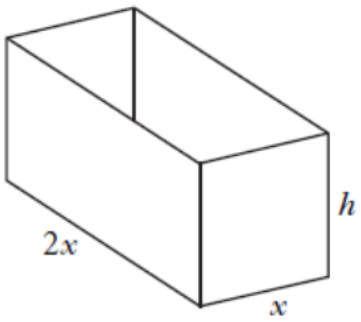
$$A = \left(10\sqrt{2}\right)x - 2x^2.$$ **3**

(*b*) Find the dimensions of the decking which maximises its area. **5**

Click here for video solution.

2004 – Paper 2 – Question 9

An open cuboid measures internally x units by $2x$ units by h units and has an inner surface area of 12 units^2.



(*a*) Show that the volume, V units^3, of the cuboid is given by $V(x) = \frac{2}{3}x(6 - x^2)$. **3**

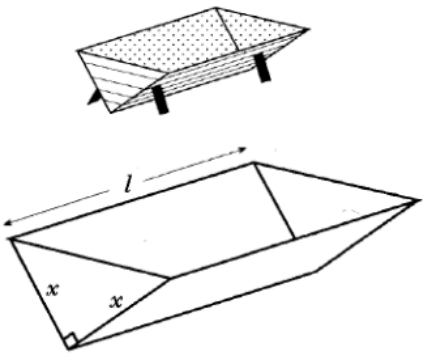
(*b*) Find the exact value of x for which this volume is a maximum. **5**

Click here for video solution.

2003 – Paper 2 – Question 8

An open water tank, in the shape of a triangular prism, has a capacity of 108 litres. The tank is to be lined on the inside in order to make it watertight.

The triangular cross-section of the tank is right-angled and isosceles, with equal sides of length x cm. The tank has a length of l cm.



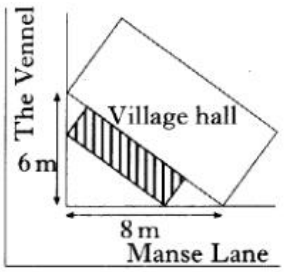
(*a*) Show that the surface area to be lined, A cm^2, is given by $A(x) = x^2 + \frac{432000}{x}$. **3**

(*b*) Find the value of x which minimises this surface area. **5**

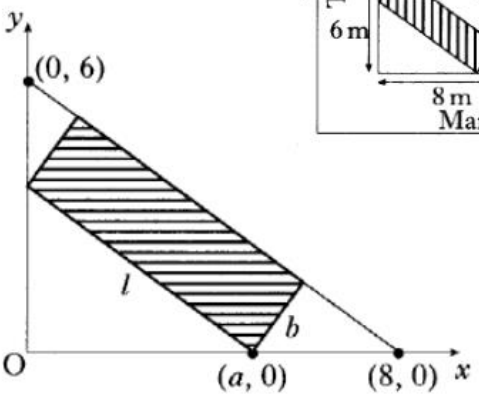
Click here for video solution.

2002 – Paper 2 – Question 10

The shaded rectangle on this map represents the planned extension to the village hall. It is hoped to provide the largest possible area for the extension.



The coordinate diagram represents the right angled triangle of ground behind the hall. The extension has length l metres and breadth b metres, as shown. One corner of the extension is at the point $(a, 0)$.



(a) (i) Show that $l = \frac{5}{4}a$.

(ii) Express b in terms of a and hence deduce that the area, A m^2, of the extension is given by $A = \frac{3}{4}a(8-a)$. **3**

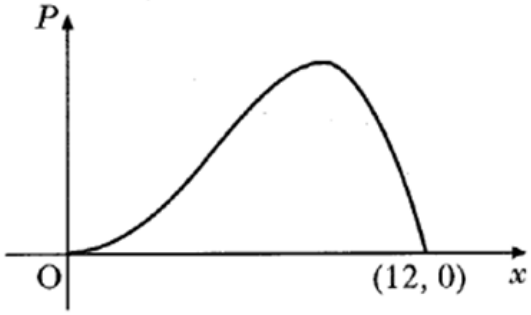
(b) Find the value of a which produces the largest area of the extension. **4**

Click here for video solution.

2001 – Paper 1 – Question 6

A company spends x thousand pounds a year on advertising and this results in a profit of P thousand pounds. A mathematical model, illustrated in the diagram, suggests that P and x are related by $P = 12x^3 - x^4$ for $0 \le x \le 12$.

Find the value of x which gives the maximum profit.



5

Click here for video solution.