Outcome 3 – Inconsistent equations

Worked Example:

Solve the system of equations

$$x + 2y + 4z = 5$$
$$3x + 3y + 10z = 19$$
$$2x + y + 6z = 16.$$

1. Solve using Gaussian elimination

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & 5 \\ 3 & 3 & 10 & 19 \\ 2 & 1 & 6 & 16 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & 5 \\ 0 & -3 & -2 & 4 \\ 0 & -3 & -2 & 6 \end{array}\right] \quad \begin{array}{l} R_2 - 3R_1 \\ R_3 - 2R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & 5 \\ 0 & -3 & -2 & 4 \\ 0 & 0 & 0 & 2 \end{array}\right] \quad R_3 - R_2$$

2. Write statement

$0 = 2$ is a contradiction.

Therefore there is no solution to this system of equations and the system is inconsistent.

Key Facts/Formulae:

Gaussian elimination is a neat way to solve a system of equations with 3 variables.

"Upper triangular" form looks like this →

$$\left[\begin{array}{ccc|c} * & * & * & * \\ 0 & * & * & * \\ 0 & 0 & * & * \end{array}\right]$$

Not all systems of equations have a unique solution. Some have infinitely many solutions, whereas some have no solution at all.

When a system of equations is inconsistent then there is no solution to the system of equations.

This happens when you end up with a row of zero coefficients equalling a non-zero constant, e.g.

$$\left[\begin{array}{ccc|c} * & * & * & * \\ 0 & * & * & * \\ 0 & 0 & 0 & c \end{array}\right]$$

Questions…

Solve the following systems of equations.

1. $x + 6y + 2z = 8$
   $4x + 22y + 12z = 35$
   $2x + 10y + 8z = 26$

2. $x + 2y + 3z = 5$
   $6x + 9y + 24z = 28$
   $2x + y + 12z = 7$

3. $x + 4y + 2z = 8$
   $3x + 14y + z = 30$
   $7x + 30y + 9z = 58$

4. $x + 5y + 2z = 9$
   $10x + 53y + 24z = 91$
   $3x + 18y + 10z = 20$

5. $x - 2y + 6z = -4$
   $2x + 14z = -9$
   $3x - 2y + 20z = -10$

6. $x + 6y - 4z = 2$
   $4x + 29y - 18z = 16$
   $9x + 59y - 38z = 16$

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Answers

1. $0 = 7$ is a contradiction. Therefore there is no solution to this system of equations and the system is inconsistent.

2. $0 = -1$ is a contradiction. Therefore there is no solution to this system of equations and the system is inconsistent.

3. $0 = -4$ is a contradiction. Therefore there is no solution to this system of equations and the system is inconsistent.

4. $0 = -6$ is a contradiction. Therefore there is no solution to this system of equations and the system is inconsistent.

5. $0 = 3$ is a contradiction. Therefore there is no solution to this system of equations and the system is inconsistent.

6. $0 = -10$ is a contradiction. Therefore there is no solution to this system of equations and the system is inconsistent.

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