Outcome 2 – Redundant equations

Worked Example:

Solve the system of equations

$$x + y + z = 2$$
$$4x + 4y + 2z = 6$$
$$2x + 2y + 2z = 4.$$

1. Solve using Gaussian elimination

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 4 & 4 & 2 & 6 \\ 2 & 2 & 2 & 4 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \\ R_2 - 4R_1 \\ R_3 - 2R_1 \end{array}$$

2. Write the 3 equations

$$0 = 0$$
$$-2z = -2 \qquad z = 1$$
$$x + y + z = 2$$

3. Assign x (or y) as variable t and write the solutions

Let $x = t$ where $t \in \mathbb{R}$ $\quad x = t, \quad y = 1 - t, \quad z = 1$

Key Facts/Formulae:

Gaussian elimination is a neat way to solve a system of equations with 3 variables.

"Upper triangular" form looks like this → $\left[\begin{array}{ccc|c} * & * & * & * \\ 0 & * & * & * \\ 0 & 0 & * & * \end{array}\right]$

Not all systems of equations have a unique solution. Some have infinitely many solutions, whereas some have no solution at all.

There are an infinite number of solutions when there is a redundant equation.

(E.g. 0 = 0 is true, but isn't at all helpful!)

When this happens, we have to express our solutions in terms of another randomly chosen variable, such as t, which will be able to take any value.

Questions…

Solve the following systems of equations.

1. $$x + y + z = 7$$
$$3x + 3y + 5z = 23$$
$$2x + 2y + 2z = 14$$

2. $$x + y + z = 3$$
$$6x + 6y - 8z = 46$$
$$4x + 4y + 4z = 12$$

3. $$x + y + z = 11$$
$$x + y + 9z = 35$$
$$5x + 5y + 5z = 55$$

4. $$x + y + z = 7$$
$$2x + 2y - z = 11$$
$$3x + 3y + 3z = 21$$

5. $$x + y + z = 2$$
$$3x + 3y + 2z = 11$$
$$6x + 6y + 6z = 12$$

6. $$x + y + z = 1$$
$$5x + 5y + 2z = 61$$
$$7x + 7y + 7z = 7$$

Answers

1. $x = t, \quad y = 3 - t, \quad z = 4$

2. $x = t, \quad y = 5 - t, \quad z = -2$

3. $x = t, \quad y = 8 - t, \quad z = 3$

4. $x = t, \quad y = 6 - t, \quad z = 1$

5. $x = t, \quad y = 7 - t, \quad z = -5$

6. $x = t, \quad y = 9 - t, \quad z = 8$