Outcome 1 – Gaussian elimination

Worked Example:

Use Gaussian elimination to solve the following system of equations

$$\begin{aligned} 2x+3y+z &= 1 \\ x-2y+5z &= -28 \\ 3x+4y+2z &= -2. \end{aligned}$$

Key Facts/Formulae:

Gaussian elimination is a neat way to solve a system of equations with 3 variables.

"Upper triangular" form looks like this →

$$\left[\begin{array}{ccc|c} * & * & * & * \\ 0 & * & * & * \\ 0 & 0 & * & * \end{array}\right]$$

1. Form an 'augmented' matrix

$$\left[\begin{array}{ccc|c} 2 & 3 & 1 & 1 \\ 1 & -2 & 5 & -28 \\ 3 & 4 & 2 & -2 \end{array}\right]$$

2. Get matrix in "upper triangular" form

$$\left[\begin{array}{ccc|c} 2 & 3 & 1 & 1 \\ 0 & -7 & 9 & -57 \\ 0 & -1 & 1 & -7 \end{array}\right] \begin{array}{l} 2R_2 - R_1 \\ 2R_3 - 3R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 1 & 1 \\ 0 & -7 & 9 & -57 \\ 0 & 0 & -2 & 8 \end{array}\right] 7R_3 - R_2$$

3. Solve for z

$-2z = 8 \qquad z = -4$

4. Solve for y and x

$-7y - 36 = -57$

$-7y = -21$

$y = 3$

$2x + 9 - 4 = 1$

$2x + 5 = 1$

$2x = -4$

$x = -2$

5. State final solutions and check solutions (optional)

$x = -2, \quad y = 3, \quad z = -4$

Questions…

Use Gaussian elimination to solve the following systems of equations.

1. $\begin{aligned} 2x+y+4z &= 16 \\ x-3y+2z &= 1 \\ x+5y-2z &= 5 \end{aligned}$

2. $\begin{aligned} 3x-2y+z &= 15 \\ x+5y-4z &= -18 \\ 4x+y+3z &= 23 \end{aligned}$

3. $\begin{aligned} 4x+2y-3z &= -1 \\ x-5y+z &= -26 \\ 2x+3y-4z &= 7 \end{aligned}$

4. $\begin{aligned} x+8y-2z &= 10 \\ 2x+10y-z &= 5 \\ x+y+z &= -6 \end{aligned}$

5. $\begin{aligned} 2x+3y+4z &= -20 \\ 4x-5y+2z &= 0 \\ x+y+z &= -6 \end{aligned}$

6. $\begin{aligned} 4x+y-3z &= 7 \\ 2x+4y-z &= 10 \\ 6x-2y+5z &= -6 \end{aligned}$

Answers

1 $x = 1, \quad y = 2, \quad z = 3$

2 $x = 3, \quad y = -1, \quad z = 4$

3 $x = -2, \quad y = 5, \quad z = 1$

4 $x = -4, \quad y = 1, \quad z = -3$

5 $x = -1, \quad y = -2, \quad z = -3$

6 $x = \frac{1}{2}, \quad y = 2, \quad z = -1$