Outcome 2 – Finding integers 'a' and 'b'

Worked Example:

Use the Euclidean algorithm to find integers a and b such that $345a + 782b = 23$.

1. Use the Euclidean algorithm to confirm the g.c.d.

$782 = 2 \times 345 + 92$
$345 = 3 \times 92 + 69$
$92 = 1 \times 69 + 23$
$69 = 3 \times 23 + 0$

$\gcd(345, 782) = 23$

2. Rearrange the expressions (excluding the last one) to make the remainders the subjects of the formulae.

$23 = 92 - 1 \times 69$
$69 = 345 - 3 \times 92$
$92 = 782 - 2 \times 345$

$23 = 92 \times 1 - 1 \times 69$
$23 = 92 \times 1 - 1(345 - 3 \times 92)$
$23 = 92 \times 1 - 1 \times 345 + 3 \times 92$
$23 = 4 \times 92 - 1 \times 345$
$23 = 4 \times (782 - 2 \times 345) - 1 \times 345$
$23 = 4 \times 782 - 8 \times 345 - 1 \times 345$
$23 = 4 \times 782 - 9 \times 345$

3. Back substitute until the integers are found.
4. Answer the question.

$345a + 782b = 23$ when $a = -9$ and $b = 4$.

Key Facts/Formulae:

The greatest common divisor (g.c.d.) of two integers is the largest integer that divides into both of them with no remainder.

This was previously called the highest common factor (h.c.f.) way back in 3rd Level Maths!

The Euclidean algorithm is a tidy method to help us find the greatest common divisor (or g.c.d.) of two integers without having to list all the divisors of each number.

Questions...

Use the Euclidean algorithm to find two integers that satisfy the equations below;

1. $609a + 189b = 21$
2. $2542p + 574q = 82$
3. $407a + 1554b = 37$
4. $1530p + 2193q = 51$
5. $1037a + 3477b = 61$
6. $8658p + 1950q = 78$

Answers

1 $a = -4$ and $b = 13$

2 $p = -2$ and $q = 9$

3 $a = -19$ and $b = 5$

4 $p = -10$ and $q = 7$

5 $a = -10$ and $b = 3$

6 $p = -9$ and $q = 40$