Outcome 3 – Finding specific terms in a binomial expansion

Worked Example:

Find the general term of the binomial expansion of $\left(3x + \dfrac{5}{x^2}\right)^{12}$ and hence find the term independent of x.

1. Find the general term using the formula
2. Simplify the general term using laws of indices
3. Calculate 'r'
4. Calculate the specific term

$$\binom{n}{r} a^{n-r} b^r = \binom{12}{r} (3x)^{12-r} \left(\frac{5}{x^2}\right)^r$$

$$= \binom{12}{r} 3^{12-r} x^{12-r} \, 5^r x^{-2r}$$

$$= \binom{12}{r} 3^{12-r} \, 5^r \, x^{12-3r}$$

12 → Shift → ÷ → 8

For the term independent of x,

$$12 - 3r = 0$$
$$3r = 12$$
$$r = 4$$

$$\binom{12}{4} 3^8 \, 5^4 \, x^0$$

$$= 495 \times 6561 \times 625 \times 1$$

$$= 2\,029\,809\,375$$

Key Facts/Formulae:

The Binomial Theorem is a quick way of multiplying out brackets with larger powers.

You will need to know Pascals triangle in order to do this.

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            1
          1   1
        1   2   1
      1   3   3   1
    1   4   6   4   1
  1   5  10  10   5   1
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Advanced Higher Formula sheet

Binomial Theorem

$$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$$

Given on the formula sheet!

where $\binom{n}{r} = {}^nC_r = \dfrac{n!}{r!\,(n-r)!}$

Questions…

1. Find the general term of the binomial expansion of $\left(3x + \dfrac{5}{x^2}\right)^6$ and hence find the term independent of x.

2. Find the general term of the binomial expansion of $\left(2x^2 + \dfrac{6}{x}\right)^9$ and hence find the term in x^9.

3. Find the general term of the binomial expansion of $\left(x^2 + \dfrac{1}{x^4}\right)^8$ and hence find the term in x^4.

4. Find the general term of the binomial expansion of $\left(2x + \dfrac{4}{x^3}\right)^{10}$ and hence find the term in x^2.

5. Find the general term of the binomial expansion of $\left(6x^3 + \dfrac{2}{x^2}\right)^{12}$ and hence find the term in x.

6. Find the general term of the binomial expansion of $\left(8x - \dfrac{3}{x^2}\right)^7$ and hence find the term in $\dfrac{1}{x^2}$.

Answers

1 General term $= \binom{6}{r} 3^{6-r} 5^r x^{6-3r}$

Term independent of $x = 30\ 375$

2 General term $= \binom{9}{r} 2^{9-r} 6^r x^{18-3r}$

Term in x^9 is $1\ 161\ 216 x^9$

3 General term $= \binom{8}{r} x^{16-6r}$

Term in x^4 is $28 x^4$

4 General term $= \binom{10}{r} 2^{10-r} 4^r x^{10-4r}$

Term in x^2 is $184\ 320 x^2$

5 General term $= \binom{12}{r} 6^{12-r} 2^r x^{36-5r}$

Term in x is $788\ 299\ 776 x$

6 General term $= \binom{7}{r} 8^{7-r} (-3)^r x^{7-3r}$

Term in $\frac{1}{x^2}$ is $-\frac{30\ 965\ 760}{x^2}$