Outcome 1 – de Moivre's theorem

Worked Example:

Given that $z = 2(\cos 210° + i \sin 210°)$, express z^4 in the form $a + bi$ where a and b are real numbers.

1. Apply de Moivre's theorem

$$z^4 = 16(\cos 840° + i \sin 840°)$$

$$z^4 = 16\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$$

$$z^4 = -8 + 8\sqrt{3}i$$

2. Evaluate the exact values

$$\cos 840° = \cos 120° = -\cos 60° = -\frac{1}{2}$$

$$\sin 840° = \sin 120° = \sin 60° = \frac{\sqrt{3}}{2}$$

Key Facts/Formulae:

To multiply two complex numbers, in polar form, e.g. $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$...

- multiply the moduli
- add the arguments

$$z_1 z_2 = r_1 r_2(\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2))$$

Consider $z = r(\cos\theta + i\sin\theta)$.

$$z^2 = r^2(\cos 2\theta + i\sin 2\theta)$$

$$z^3 = r^3(\cos 3\theta + i\sin 3\theta)$$

$$z^n = r^n(\cos n\theta + i\sin n\theta)$$

Given on formula sheet!

Questions...

1. Given that $z = 10(\cos 60° + i \sin 60°)$, express z^2 in the form $a + bi$ where a and b are real numbers.

2. Given that $z = 6(\cos 90° + i \sin 90°)$, express z^3 in the form $a + bi$ where a and b are real numbers.

3. Given that $z = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$, express z^4 in the form $a + bi$ where a and b are real numbers.

4. Given that $z = 8(\cos 240° + i \sin 240°)$, express z^2 in the form $a + bi$ where a and b are real numbers.

5. Given that $z = 3\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$, express z^5 in the form $a + bi$ where a and b are real numbers.

6. Given that $z = 36(\cos 300° + i \sin 300°)$, express $\sqrt{z}$ in the form $a + bi$ where a and b are real numbers.

Answers

1 $-50 + 50\sqrt{3}i$

2 $-216i$

3 $-16\sqrt{3} + 16i$

4 $-32 + 32\sqrt{3}i$

5 $-\frac{243}{\sqrt{2}} - \frac{243}{\sqrt{2}}i$

6 $-3\sqrt{3} + 3i$