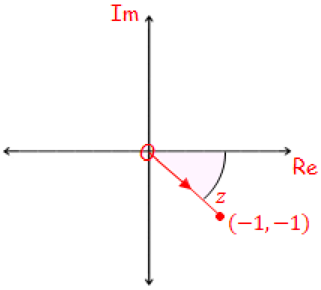
Outcome 3 – The geometry of complex numbers in the 4th quadrant

Worked Example:

Given that $z = 1 - i$

(a) sketch on an argand diagram.



Calculate;

(b) $|z|$ $|z| = \sqrt{1+1} = \sqrt{2}$ $\tan^{-1}\frac{1}{1} = 45°$

(c) $\arg z$ $\arg z = -45°$ $\left(-\frac{\pi}{4}\right)$

(d) Express the complex number z in polar form.

$z = \sqrt{2}(\cos(-45°) + i\sin(-45°))$

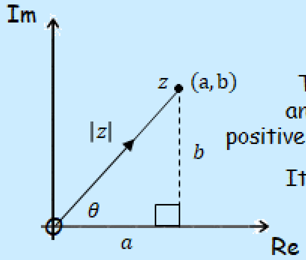
$\left[z = \sqrt{2}\left(\cos(-\frac{\pi}{4}) + i\sin(-\frac{\pi}{4})\right)\right]$

Key Facts/Formulae:

The complex number $z = a + bi$ can be represented in an argand diagram.

The modulus of z is the distance from the origin to z and is denoted by $|z|$ or r.

$|z| = \sqrt{a^2 + b^2}$



The argument of z is the angle between Oz and the positive direction of the x-axis.

It is denoted by $\arg z$ or θ and lies between $-180° < \theta < 180°$.

Polar Form $\cos\theta = \frac{a}{r}$ $a = r\cos\theta$ $\theta = \tan^{-1}\frac{b}{a}$

$z = a + bi$ $\sin\theta = \frac{b}{r}$ $b = r\sin\theta$

$z = r\cos\theta + ir\sin\theta$

$z = r(\cos\theta + i\sin\theta)$

None of these are on the formula sheet!

Questions...

For each complex number below;

(a) Express in an argand diagram
(b) Calculate the modulus
(c) Calculate the argument
(d) Write in polar form

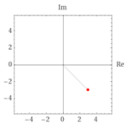
1 $z = 3 - 3i$

2 $z = 4 - 4\sqrt{3}i$

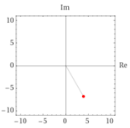
3 $z = 10\sqrt{3} - 10i$

4 $z = -5i$

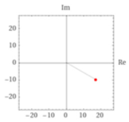
Answers

1 (a) 

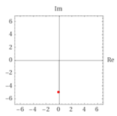
(b) $|z| = 3\sqrt{2}$ (c) $\arg z = -45°$ (d) $z = 3\sqrt{2}(\cos(-45°) + i\sin(-45°))$ (or $-\frac{\pi}{4}$ if using radians)

2 (a) 

(b) $|z| = 8$ (c) $\arg z = -60°$ (d) $z = 8(\cos(-60°) + i\sin(-60°))$ (or $-\frac{\pi}{3}$ if using radians)

3 (a) 

(b) $|z| = 20$ (c) $\arg z = -30°$ (d) $z = 20(\cos(-30°) + i\sin(-30°))$ (or $-\frac{\pi}{6}$ if using radians)

4 (a) 

(b) $|z| = 5$ (c) $\arg z = -90°$ (d) $z = 5(\cos(-90°) + i\sin(-90°))$ (or $-\frac{\pi}{2}$ if using radians)