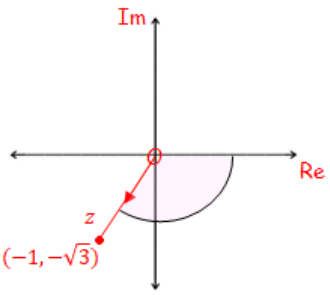
Outcome 4 – The geometry of complex numbers in the 3rd quadrant

Worked Example:

Given that $z = -1 - \sqrt{3}i$

(a) sketch on an argand diagram.



Calculate;

(b) $|z|$ $\quad |z| = \sqrt{1+3} = \sqrt{4} = 2$ $\quad \tan^{-1}\frac{\sqrt{3}}{1} = 60°$

$180 - 60 = 120°$

(c) $\arg z$ $\quad \arg z = -120° \left(-\frac{2\pi}{3}\right)$

(d) Express the complex number z in polar form.

$z = 2(\cos(-120°) + i\sin(-120°))$

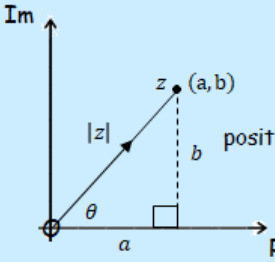
$\left[z = 2\left(\cos(-\frac{2\pi}{3}) + i\sin(-\frac{2\pi}{3})\right)\right]$

Key Facts/Formulae:

The complex number $z = a + bi$ can be represented in an argand diagram.

The modulus of z is the distance from the origin to z and is denoted by $|z|$ or r.

$|z| = \sqrt{a^2 + b^2}$



The argument of z is the angle between Oz and the positive direction of the x-axis.

It is denoted by $\arg z$ or θ and lies between $-180° < \theta < 180°$.

Polar Form

$\cos\theta = \frac{a}{r}$ $\quad a = r\cos\theta$ $\quad \theta = \tan^{-1}\frac{b}{a}$

$z = a + bi$ $\quad \sin\theta = \frac{b}{r}$ $\quad b = r\sin\theta$

$z = r\cos\theta + ir\sin\theta$

$z = r(\cos\theta + i\sin\theta)$

None of these are on the formula sheet!

Questions…

For each complex number below;

(a) Express in an argand diagram
(b) Calculate the modulus
(c) Calculate the argument
(d) Write in polar form

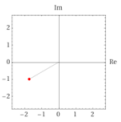
1. $z = -\sqrt{3} - i$

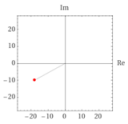
2. $z = -8 - 8\sqrt{3}i$

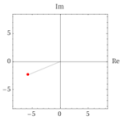
3. $z = -\sqrt{11} - \sqrt{11}i$

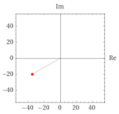
4. $z = -20\sqrt{3} - 20i$

Answers

1 (a)  (b) $|z| = 2$ (c) $\arg z = -150°$ (d) $z = 2(\cos(-150°) + i\sin(-150°))$ (or $-\frac{5\pi}{6}$ if using radians)

2 (a)  (b) $|z| = 16$ (c) $\arg z = -120°$ (d) $z = 16(\cos(-120°) + i\sin(-120°))$ (or $-\frac{2\pi}{3}$ if using radians)

3 (a)  (b) $|z| = \sqrt{22}$ (c) $\arg z = -135°$ (d) $z = \sqrt{22}(\cos(-135°) + i\sin(-135°))$ (or $-\frac{3\pi}{4}$ if using radians)

4 (a)  (b) $|z| = 40$ (c) $\arg z = -150°$ (d) $z = 40(\cos(-150°) + i\sin(-150°))$ (or $-\frac{5\pi}{6}$ if using radians)