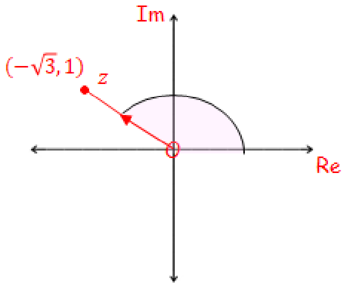
Outcome 2 – The geometry of complex numbers in the 2nd quadrant

Worked Example:

Given that $z = -\sqrt{3} + i$

(a) sketch on an argand diagram.



Calculate;

(b) $|z|$ $\quad |z| = \sqrt{3+1} = \sqrt{4} = 2 \quad \tan^{-1}\frac{1}{\sqrt{3}} = 30°$

(c) $\arg z$ $\quad \arg z = 180 - 30 = 150° \left(\frac{5\pi}{6}\right)$

(d) Express the complex number z in polar form.

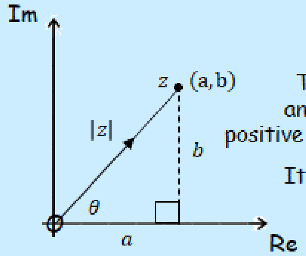
$$z = 2(\cos 150° + i\sin 150°)$$

$$\left[z = 2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)\right]$$

Key Facts/Formulae:

The complex number $z = a + bi$ can be represented in an argand diagram.

The modulus of z is the distance from the origin to z and is denoted by $|z|$ or r.



$$|z| = \sqrt{a^2 + b^2}$$

The argument of z is the angle between Oz and the positive direction of the x-axis.

It is denoted by $\arg z$ or θ and lies between $-180° < \theta < 180°$.

Polar Form

$\cos\theta = \frac{a}{r}$ $\quad a = r\cos\theta$ $\quad \theta = \tan^{-1}\frac{b}{a}$

$z = a + bi$

$\sin\theta = \frac{b}{r}$ $\quad b = r\sin\theta$

$z = r\cos\theta + ir\sin\theta$

$z = r(\cos\theta + i\sin\theta)$

None of these are on the formula sheet!

Questions…

For each complex number below;

(a) Express in an argand diagram
(b) Calculate the modulus
(c) Calculate the argument
(d) Write in polar form

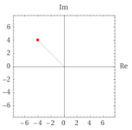
1. $z = -4 + 4i$

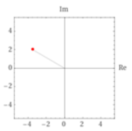
2. $z = -2\sqrt{3} + 2i$

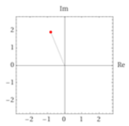
3. $z = -\sqrt{7} + \sqrt{7}i$

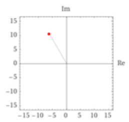
4. $z = -6 + 6\sqrt{3}i$

Answers

1 (a)  (b) $|z| = 4\sqrt{2}$ (c) $\arg z = 135°$ (d) $z = 4\sqrt{2}(\cos 135° + i \sin 135°)$ (or $\frac{3\pi}{4}$ if using radians)

2 (a)  (b) $|z| = 4$ (c) $\arg z = 150°$ (d) $z = 4(\cos 150° + i \sin 150°)$ (or $\frac{5\pi}{6}$ if using radians)

3 (a)  (b) $|z| = \sqrt{14}$ (c) $\arg z = 135°$ (d) $z = \sqrt{14}(\cos 135° + i \sin 135°)$ (or $\frac{3\pi}{4}$ if using radians)

4 (a)  (b) $|z| = 12$ (c) $\arg z = 120°$ (d) $z = 12(\cos 120° + i \sin 120°)$ (or $\frac{2\pi}{3}$ if using radians)