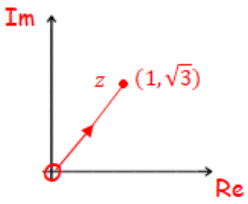
# Outcome 1 – The geometry of complex numbers in the 1st quadrant

## Worked Example:

Given that $z = 1 + \sqrt{3}i$




(a) sketch on an argand diagram.

Calculate;

(b) $|z|$ $\quad |z| = \sqrt{1+3} = \sqrt{4} = 2$

(c) $\arg z$ $\quad \arg z = \tan^{-1}\frac{\sqrt{3}}{1} = 60° \quad \left(\frac{\pi}{3}\right)$

(d) Express the complex number $z$ in polar form.

$z = 2(\cos 60° + i\sin 60°)$

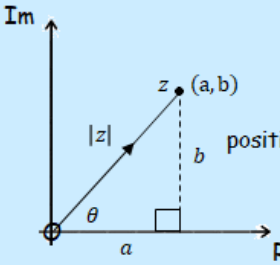
$\left[z = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right]$

**Key Facts/Formulae:**

The complex number $z = a + bi$ can be represented in an argand diagram.

The modulus of $z$ is the distance from the origin to $z$ and is denoted by $|z|$ or r.

$|z| = \sqrt{a^2 + b^2}$




The argument of $z$ is the angle between Oz and the positive direction of the x-axis.

It is denoted by $\arg z$ or $\theta$ and lies between $-180° < \theta < 180°$.

Polar Form

$z = a + bi$

$z = r\cos\theta + ir\sin\theta$

$z = r(\cos\theta + i\sin\theta)$

$\cos\theta = \frac{a}{r}$ $\quad a = r\cos\theta$

$\sin\theta = \frac{b}{r}$ $\quad b = r\sin\theta$

$\theta = \tan^{-1}\frac{b}{a}$

None of these are on the formula sheet!

## Questions…

For each complex number below;

(a) Express in an argand diagram
(b) Calculate the modulus
(c) Calculate the argument
(d) Write in polar form

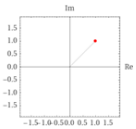
1. $z = 1 + i$

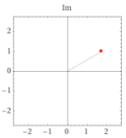
2. $z = \sqrt{3} + i$

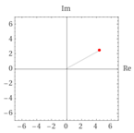
3. $z = 2 + 2\sqrt{3}i$

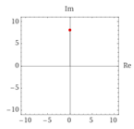
4. $z = 8i$

# Answers

1 (a)  (b) $|z| = \sqrt{2}$ (c) $\arg z = 45°$ (d) $z = \sqrt{2}(\cos 45° + i \sin 45°)$ (or $\frac{\pi}{4}$ if using radians)

2 (a)  (b) $|z| = 2$ (c) $\arg z = 30°$ (d) $z = 2(\cos 30° + i \sin 30°)$ (or $\frac{\pi}{6}$ if using radians)

3 (a)  (b) $|z| = 4$ (c) $\arg z = 60°$ (d) $z = 4(\cos 60° + i \sin 60°)$ (or $\frac{\pi}{3}$ if using radians)

4 (a)  (b) $|z| = 8$ (c) $\arg z = 90°$ (d) $z = 8(\cos 90° + i \sin 90°)$ (or $\frac{\pi}{2}$ if using radians)