Outcome 2 – Solving cubic equations

Key Facts/Formulae:

We can find roots using;

- Synthetic division
- The quadratic formula
- The conjugate roots property

We can find factors by dividing!

Essential knowledge!

Complex roots of a polynomial equation (with real coefficients) occur in conjugate pairs.

E.g. if $a+bi$ is a root, then so is $a-bi$.

Worked Example:

Given that $z = 3+2i$ is a root of the equation $z^3 - 8z^2 + 25z - 26 = 0$,

(a) Write down another root of this equation
(b) Find all the roots of the equation

1. Write down another root

$z = 3-2i$

2. Form two factors

If $z = 3+2i$ is a root then $z-(3+2i)$ is a factor.

If $z = 3-2i$ is a root then $z-(3-2i)$ is a factor.

$$\begin{array}{r|l} z^2-6z+13 & z-2 \\ & z^3-8z^2+25z-26 \\ & -z^3-6z^2+13z \\ \hline & -2z^2+12z-26 \\ & -2z^2+12z-26 \\ \hline & 0 \end{array}$$

2. Multiply the two factors to form a trinomial

$$\{(z-3)-2i\}\{(z-3)+2i\} = z^2-6z+9+2zi-6i-2zi+6i+4 = z^2-6z+13$$

3. Use algebraic long division to find the other root and state all roots

$z = 3+2i$ $\quad z = 3-2i$ $\quad z = 2$

Questions…

1. Given that $z = 2+i$ is a root of the equation $z^3 - 7z^2 + 17z - 15 = 0$,

(a) Write down another root of this equation
(b) Find all the roots of the equation

2. Given that $z = 7+2i$ is a root of the equation $z^3 + 13z^2 + 22z + 100 = 0$,

(a) Write down another root of this equation
(b) Find all the roots of the equation

3. Given that $z = 2-8i$ is a root of the equation $z^3 - 9z^2 + 88z - 340 = 0$,

(a) Write down another root of this equation
(b) Find all the roots of the equation

4. Given that $z = 3+4i$ is a root of the equation $z^3 + 3z^2 - 29z + 225 = 0$,

(a) Write down another root of this equation
(b) Find all the roots of the equation

5. Given that $z = -1+3i$ is a root of the equation $z^3 - 7z^2 + 2z - 40 = 0$,

(a) Write down another root of this equation
(b) Find all the roots of the equation

6. Given that $z = -6-2i$ is a root of the equation $z^3 + 22z^2 + 160z + 400 = 0$,

(a) Write down another root of this equation
(b) Find all the roots of the equation

Answers

1 $z = 2 + i$, $z = 2 - i$, $z = 3$

2 $z = 7 + i$, $z = 7 - i$, $z = -2$

3 $z = 2 + 8i$, $z = 2 - 8i$, $z = 5$

4 $z = 3 + 4i$, $z = 3 - 4i$, $z = -9$

5 $z = -1 + 3i$, $z = -1 - 3i$, $z = 4$

6 $z = -6 + 2i$, $z = -6 - 2i$, $z = -10$