Key Facts/Formulae:

We can find roots using;

· Synthetic division

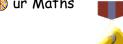
Essential knowledge!

The quadratic formulaThe conjugate roots property

Complex roots of a polynomial equation (with

real coefficients) occur in conjugate pairs. E.g. if a + bi is a root, then so is a - bi.





We can find

factors by

dividing!

Outcome 2 - Solving cubic equations

Worked Example:

Given that z = 3 + 2i is a root of the equation $z^3 - 8z^2 + 25z - 26 = 0$,

- (a) Write down another root of this equation
- (b) Find all the roots of the equation

1. Write down another root

$$z = 3 - 2i$$

2. Form two factors

If z = 3 + 2i is a root then z - (3 + 2i) is a factor. If z = 3 - 2i is a root then z - (3 - 2i) is a factor.

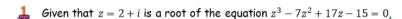
2. Multiply the two factors to form a trinomial

$\{(z-3)-2i\}\{(z-3)+2i\}=z^2-6z+9+2(i-6i-2)zi+6i+4=z^2-6z+13$

3. Use algebraic long division to find the other root and state all roots

$$z = 3 + 2i$$
 $z = 3 - 2i$ $z = 2i$

Questions...



- (a) Write down another root of this equation
- (b) Find all the roots of the equation

Given that
$$z = 7 + 2i$$
 is a root of the equation $z^3 + 13z^2 + 22z + 100 = 0$,

- (a) Write down another root of this equation
 - (b) Find all the roots of the equation

Given that
$$z = 2 - 8i$$
 is a root of the equation $z^3 - 9z^2 + 88z - 340 = 0$,

- (a) Write down another root of this equation
- (b) Find all the roots of the equation

Given that
$$z = 3 + 4i$$
 is a root of the equation $z^3 + 3z^2 - 29z + 225 = 0$,

- (a) Write down another root of this equation
- (b) Find all the roots of the equation

Given that
$$z = -1 + 3i$$
 is a root of the equation $z^3 - 7z^2 + 2z - 40 = 0$,

- (a) Write down another root of this equation
- (b) Find all the roots of the equation

Given that
$$z = -6 - 2i$$
 is a root of the equation $z^3 + 22z^2 + 160z + 400 = 0$,

- (a) Write down another root of this equation
- (b) Find all the roots of the equation

Answers

$$z = 2 + i,$$
 $z = 7 + i,$ $z = 7 - i,$ $z = 3$ $z = 7 - i,$

$$z = 7 + i,$$

$$z = 7 - i,$$

$$z = -2$$

$$z = 2 + 8i,$$
 $z = 3 + 4i,$ $z = 3 - 4i,$ $z = 5$ $z = -9$

$$z = 3 + 4i,$$

$$z = 3 - 4i,$$

$$z = -9$$

$$z = -1 + 3i,$$

$$z = -1 - 3i,$$

$$z = 4$$

$$z = -1 + 3i, z = -6 + 2i, z = -6 - 2i, z = 4$$

$$z = -6 - 2i, z = -10$$