Outcome 4 – Square rooting complex numbers

Worked Example:

Find the square root of $\sqrt{7+24i}$ in the form $a+bi$.

Essential knowledge!

$i^2 = -1 \qquad i^3 = -i \qquad i^4 = 1$

1. Set equal to $a+bi$

$$a+bi = \sqrt{7+24i}$$

2. Square both sides

$$(a+bi)^2 = 7+24i$$

$$a^2 + 2abi - b^2 = 7+24i$$

3. Compare the real and imaginary parts

$$a^2 - b^2 + 2abi = 7+24i$$

$a^2 - b^2 = 7 \qquad 2ab = 24$

$ab = 12$

$b = \frac{12}{a}$

$a^2 - \frac{144}{a^2} = 7$

$a^4 - 144 = 7a^2$

$-7a^2 - 144 = 0$

$(a^2+9)(a^2-16) = 0$

~~$a^2 = -9$~~ $\qquad a^2 = 16$

$a = 4, -4$

$b = 3, -3$

$4+3i, \qquad -4-3i$

Key Facts/Formulae:

i, the imaginary number, is defined as $i = \sqrt{-1}$

A complex number, z, is one that can be written in the form $a+bi$.

a is the real part $\qquad b$ is the imaginary part

To add/subtract complex numbers;

- add/subtract the real parts
- add/subtract the imaginary parts

To multiply complex numbers;

- form and multiply out brackets

Questions…

Find the square root of the following in the form $a+bi$;

1. $\sqrt{-1+4i}$
2. $\sqrt{-5+12i}$
3. $\sqrt{15+8i}$
4. $\sqrt{32+24i}$
5. $\sqrt{9+40i}$
6. $\sqrt{75+100i}$

Answers

1 $1 + 2i$ and $-1 - 2i$

2 $2 + 3i$ and $-2 - 3i$

3 $4 + i$ and $-4 - i$

4 $6 + 2i$ and $-6 - 2i$

5 $5 + 4i$ and $-5 - 4i$

6 $10 + 5i$ and $-10 - 5i$