Outcome 1 – Separable first order differential equations

(a) Find the general solution of the differential equation

$$\frac{dy}{dx} = \frac{x^4}{y}$$

(b) Hence find the particular solution given that $y = 3$ and $x = 1$.

1. Separate the 'x's and 'y's
2. Integrate both sides

(a) $y\,dy = x^4\,dx$

$$\int y\,dy = \int x^4\,dx$$

$$\frac{y^2}{2} = \frac{x^5}{5} + c$$

× 10) $5y^2 = 2x^5 + c$

3. Sub in the conditions

(b) When $y = 3$ and $x = 1$,

$$45 = 2 + c$$

$$c = 43$$

$$5y^2 = 2x^5 + 43$$

Key Facts/Formulae:

A differential equation is an equation containing one or more derivatives.

A first order differential equation contains a first derivative only.

The General Solution of a differential equation contains a constant of integration.

When the constant of integration is calculated by substituting a given set of conditions, the solution is then called a Particular Solution.

A differential equation is separable if it can be rearranged into the form;

$$f(y)dy = g(x)dx$$

Questions...

Find the general and then particular solution to each of these differential equations.

1. $\frac{dy}{dx} - 4x = 0$ given that $y = 10$ and $x = 3$.

2. $\frac{dy}{dx} = \frac{4x^3}{3y^2}$ given that $y = 1$ and $x = 2$.

3. $3y\frac{dy}{dx} - 2x^2 = 0$ given that $y = 3$ and $x = 1$.

4. $\frac{dy}{dx} = \frac{x^9}{y}$ given that $y = 3$ and $x = 1$.

5. $x\frac{dy}{dx} = 1$ given that $y = 5$ and $x = 1$.

6. $(1 + x^2)\frac{dy}{dx} = 1$ given that $y = \pi$ and $x = 1$.

Answers

1 $y = 2x^2 - 8$

2 $y^3 = x^4 - 15$

3 $9y^2 = 4x^3 - 72$

4 $5y^2 = x^{10} + 35$

5 $y = \ln x + 5$

6 $y = \tan^{-1} x + \frac{3\pi}{4}$