Outcome 3 – Integration by parts using the integral I

Key Facts/Formulae:

Integration by parts is used to integrate products when integration by substitution does not work.

Often, you will be told what method to use in the exam.

To integrate by parts;

1. One of your functions must be "easy" to differentiate and gets "simpler" when you differentiate it.
2. One of your functions must be "easy" to integrate

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

Use integration by parts to find $\int e^x \sin x\, dx$.

1. Define your integral 2. Choose u and $\frac{dv}{dx}$

Let $I = \int e^x \sin x\, dx$ Let $u = e^x$ and $\frac{dv}{dx} = \sin x$

3. Calculate $\frac{du}{dx}$ and v $\frac{du}{dx} = e^x$ $v = -\cos x$

4. Sub into integration by parts formula

$I = -e^x \cos x - \int -e^x \cos x\, dx$ Let $u = e^x$ and $\frac{dv}{dx} = \cos x$

$I = -e^x \cos x + \int e^x \cos x\, dx$ $\frac{du}{dx} = e^x$ $v = \sin x$

$I = -e^x \cos x + e^x \sin x - \int e^x \sin x\, dx$

$I = -e^x \cos x + e^x \sin x - I$

$2I = -e^x \cos x + e^x \sin x$ $I = \frac{1}{2} e^x (\sin x - \cos x) + C$

Questions…

Calculate each of the following integrals using integration by parts.

1. $\int e^{3x} \cos x\, dx$
2. $\int e^x \cos 4x\, dx$
3. $\int e^{3x} \cos 2x\, dx$
4. $\int \sin x \cos 2x\, dx$
5. $\int \cos x \cos 5x\, dx$
6. $\int e^{-x} \sin x\, dx$

Answers

1. $\frac{1}{5}e^{3x}(\sin x + 3\cos x) + c$

2. $\frac{1}{17}e^{x}(4\sin 4x + \cos 4x) + c$

3. $\frac{1}{13}e^{3x}(2\sin 2x + 3\cos 2x) + c$

4. $\frac{1}{3}(\cos x \cos 2x + 2\sin x \sin 2x) + c$

5. $\frac{1}{24}(5\cos x \sin 5x - \sin x \cos 5x) + c$

6. $-\frac{1}{2}e^{-x}(\cos x + \sin x) + c$