Outcome 2 – Integration by parts more than once

Worked Example:

Use integration by parts to find $\int x^2 \sin 4x \, dx$.

1. Choose u and $\frac{dv}{dx}$

Let $u = x^2$ and $\frac{dv}{dx} = \sin 4x$

2. Calculate $\frac{du}{dx}$ and v

$\frac{du}{dx} = 2x \quad v = -\frac{1}{4}\cos 4x$

3. Sub into integration by parts formula

$$\int x^2 \sin 4x \, dx = -\frac{1}{4}x^2 \cos 4x - \int -\frac{1}{2} x \cos 4x \, dx$$

Let $u = -\frac{1}{2}x$ and $\frac{dv}{dx} = \cos 4x$

$\frac{du}{dx} = -\frac{1}{2} \quad v = \frac{1}{4}\sin 4x$

$$= -\frac{1}{4}x^2 \cos 4x - \left[-\frac{1}{8} x \sin 4x - \int -\frac{1}{8}\sin 4x \, dx\right] = -\frac{1}{4}x^2 \cos 4x - \left[-\frac{1}{8}x \sin 4x - \frac{1}{32}\cos 4x\right]$$

4. Use integration by parts again!

$$= -\frac{1}{4}x^2 \cos 4x + \frac{1}{8}x \sin 4x + \frac{1}{32}\cos 4x + c$$

Key Facts/Formulae:

Integration by parts is used to integrate products when integration by substitution does not work.

Often, you will be told what method to use in the exam.

To integrate by parts;

1. One of your functions must be "easy" to differentiate and gets "simpler" when you differentiate it.
2. One of your functions must be "easy" to integrate

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Questions...

Calculate each of the following integrals using integration by parts.

1. $\int x^2 \cos x \, dx$

2. $\int x^2 \sin 5x \, dx$

3. $\int 6x^2 \cos 9x \, dx$

4. $\int (x^2 - 2x + 5)\, e^{4x} \, dx$

5. $\int x^3 e^x \, dx$

6. $\int 6x^2 \ln x \, dx$ **Hint** You CANNOT integrate lnx!

Answers

1 $x^2 \sin x + 2x \cos x - 2 \sin x + c$

2 $-\frac{1}{5}x^2 \cos 5x + \frac{2}{25}x \sin 5x + \frac{2}{125}\cos 5x + c$

3 $\frac{2}{3}x^2 \sin 9x + \frac{4}{27}x \cos 9x - \frac{4}{243}\sin 9x + c$

4 $\frac{1}{32}e^{4x}[8x^2 - 20x + 45] + c$

5 $x^3e^x - 3x^2e^x + 6xe^x - 6e^x + c$

6 $2x^3 \ln x - \frac{2}{3}x^3 + c$