Outcome 1 – Integration by parts

Worked Example:

Use integration by parts to find $\int x \cos 3x \, dx$.

1. Choose u and $\frac{dv}{dx}$

   Let $u = x$ and $\frac{dv}{dx} = \cos 3x$

2. Calculate $\frac{du}{dx}$ and v

   $\frac{du}{dx} = 1$ $v = \frac{1}{3} \sin 3x$

3. Sub into integration by parts formula

$$\int x \cos 3x \, dx$$

$$= x \times \frac{1}{3} \sin 3x - \int \frac{1}{3} \sin 3x \times 1 \, dx$$

$$= \frac{1}{3} x \sin 3x + \frac{1}{9} \cos 3x + c$$

Questions…

Calculate each of the following integrals using integration by parts.

1. $\int x \sin x \, dx$

2. $\int x \cos 5x \, dx$

3. $\int x \, e^x \, dx$

4. $\int 4x \sin 8x \, dx$

5. $\int 12x \cos 3x \, dx$

6. $\int 6x \, e^{2x} dx$

Answers

1 $-x\cos x + \sin x + c$

2 $\dfrac{1}{5}x\sin 5x + \dfrac{1}{25}\cos 5x + c$

3 $xe^x - e^x + c$

4 $-\dfrac{1}{2}x\cos 8x + \dfrac{1}{16}\sin 8x + c$

5 $4x\sin 3x + \dfrac{4}{3}\cos 3x + c$

6 $3xe^{2x} - \dfrac{3}{2}e^{2x} + c$