



Outcome 1 - Integration by parts

Worked Example:

Use integration by parts to find $\int x \cos 3x \, dx$.

1. Choose u and $\frac{dv}{dx}$

$$\text{Let } u = x \quad \text{and} \quad \frac{dv}{dx} = \cos 3x$$

2. Calculate $\frac{du}{dx}$ and v

$$\frac{du}{dx} = 1 \quad v = \frac{1}{3} \sin 3x$$

3. Sub into integration by parts formula

$$\begin{aligned} & \int x \cos 3x \, dx \\ &= x \times \frac{1}{3} \sin 3x - \int \frac{1}{3} \sin 3x \times 1 \, dx \\ &= \frac{1}{3} x \sin 3x + \frac{1}{9} \cos 3x + c \end{aligned}$$

Questions...

Calculate each of the following integrals using integration by parts.

1 $\int x \sin x \, dx$

2 $\int x \cos 5x \, dx$

3 $\int x e^x \, dx$

4 $\int 4x \sin 8x \, dx$

5 $\int 12x \cos 3x \, dx$

6 $\int 6x e^{2x} \, dx$

Key Facts/Formulae:



Integration by parts is used to integrate products when integration by substitution does not work.

Often, you will be told what method to use in the exam.

To integrate by parts;

1. One of your functions must be "easy" to differentiate and gets "simpler" when you differentiate it.
2. One of your functions must be "easy" to integrate

$$\int u \frac{dv}{dx} \, dx = uv - \int v \frac{du}{dx} \, dx$$

Advanced Higher Formula sheet



$f(x)$	$\int f(x) \, dx$
e^{ax}	$\frac{1}{a} e^{ax} + c$

Answers

1 $-x \cos x + \sin x + c$

2 $\frac{1}{5}x \sin 5x + \frac{1}{25} \cos 5x + c$

3 $xe^x - e^x + c$

4 $-\frac{1}{2}x \cos 8x + \frac{1}{16} \sin 8x + c$

5 $4x \sin 3x + \frac{4}{3} \cos 3x + c$

6 $3xe^{2x} - \frac{3}{2}e^{2x} + c$