Outcome 3 – Integration by substitution with an extra 'x' term

Worked Example:

Calculate $\int x(x+5)^4\,dx$ using the substitution $u = x + 5$.

$u = x + 5$

$\frac{du}{dx} = 1$

$du = dx$

$x = u - 5$

$$\int x\,u^4 du$$

$$= \int (u-5)u^4 du$$

$$= \int u^5 - 5u^4\,du$$

$$= \frac{u^6}{6} - \frac{5u^5}{5} + c$$

$$= \frac{1}{6}(x+5)^6 - (x+5)^5 + c$$

Key Facts/Formulae:

This is the integration equivalent of the Chain Rule.

You will choose a new variable, u, which will usually be a function within a function.

In the exam, this choice will often be made for you.

1. Assign your new variable
2. Find an expression for 'dx'
3. Change limits to be in terms of 'u'
4. Make your substitutions and look to simplify
5. Integrate and evaluate!
6. Express final answer in terms of 'x'

Questions...

Calculate;

1. $\int x(x+6)^2\,dx$ using the substitution $u = x + 6$

2. $\int x(x-1)^9\,dx$ using the substitution $u = x - 1$

3. $\int 2x(x+4)^6\,dx$ using the substitution $u = x + 4$

4. $\int 4x(2x-3)^8\,dx$ using the substitution $u = 2x - 3$

5. $\int 5x(10x+3)^4\,dx$ using the substitution $u = 10x + 3$

6. $\int 4x^3(x^2+1)^5\,dx$ using the substitution $u = x^2 + 1$

Answers

1 $\frac{1}{4}(x+6)^4 - 2(x+6)^3 + c$

2 $\frac{1}{11}(x-1)^{11} + \frac{1}{10}(x-1)^{10} + c$

3 $\frac{1}{4}(x+4)^8 - \frac{8}{7}(x+4)^7 + c$

4 $\frac{1}{10}(2x-3)^{10} - \frac{1}{3}(2x-3)^9 + c$

5 $\frac{1}{120}(10x+3)^6 - \frac{3}{100}(10x+3)^5 + c$

6 $\frac{2}{7}(x^2+1)^7 - \frac{1}{3}(x^2+1)^6 + c$