Outcome 2 – Integration by substitution for definite integrals

Worked Example:

Evaluate $\int_0^2 \frac{2x}{x^2+3}\,dx$ using the substitution $u = x^2 + 3$.

$u = x^2 + 3$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

When $x = 2$, $u = 4 + 3 = 7$

When $x = 0$, $u = 0 + 3 = 3$

$$\int_3^7 \frac{2x}{u}\frac{du}{2x} = \int_3^7 \frac{1}{u}\,du = \Big[\ln|u|\Big]_3^7$$

$$= (\ln 7) - (\ln 3) = \ln\frac{7}{3}$$

Advanced Higher Formula sheet

$f(x)$	$\int f(x)\,dx$
$\frac{1}{x}$	$\ln\lvert x\rvert + c$

Key Facts/Formulae:

This is the integration equivalent of the Chain Rule.

You will choose a new variable, u, which will usually be a function within a function.

In the exam, this choice will often be made for you.

1. Assign your new variable
2. Find an expression for 'dx'
3. Change limits to be in terms of 'u'
4. Make your substitutions and look to simplify
5. Integrate and evaluate!

Questions...

Evaluate;

1. $\int_0^2 3x^2(x^3-1)^3\,dx$ using the substitution $u = x^3 - 1$

2. $\int_0^1 x\sqrt{x^2+1}\,dx$ using the substitution $u = x^2 + 1$

3. $\int_0^4 \frac{2x}{x^2+9}\,dx$ using the substitution $u = x^2 + 9$

4. $\int_1^3 \frac{x+2}{x^2+4x+3}\,dx$ using the substitution $u = x^2 + 4x + 3$

5. $\int_0^{\frac{\pi}{2}} \sin^5 x \cos x\,dx$ using the substitution $u = \sin x$

6. $\int_0^{\frac{\pi}{4}} \sin x \cos^3 x\,dx$ using the substitution $u = \cos x$

Answers

1 600

2 $\frac{1}{3}\left(2\sqrt{2}-1\right)$ or $\approx 0 \cdot 609$

3 $\ln\frac{25}{9}$ or $\approx 1 \cdot 022$

4 $\ln\sqrt{3}$ or $\approx 0 \cdot 549$

5 $\frac{1}{6}$

6 $-\frac{3}{16}$