## Outcome 1 – Integration by substitution for indefinite integrals

**Worked Example:**

Calculate $\displaystyle\int 6x\,(x^2 - 7)^4\,dx$

$$= \int 6x\,u^4\,\frac{du}{2x}$$

$$= \int 3u^4\,du \quad \left(3\int u^4\,du\right)$$

$$= \frac{3u^5}{5} + c$$

$$= \frac{3}{5}\,(x^2 - 7)^5 + c$$

Let $u = x^2 - 7$

$$\frac{du}{dx} = 2x$$

$$du = 2x\,dx$$

$$\frac{du}{2x} = dx$$

**Key Facts/Formulae:**

This is the integration equivalent of the Chain Rule.

You will choose a new variable, $u$, which will usually be a function within a function.

In the exam, it will often make this choice for you.

1. Assign your new variable

2. Find an expression for 'dx'

3. Make your substitutions and look to simplify

4. Integrate!

5. Express final answer in terms of 'x'

## Questions…

Calculate each of the following integrals.

**1**  $\displaystyle\int 3x^2\,(x^3 + 8)^5\,dx$

**2**  $\displaystyle\int x\,(x^2 + 10)^3\,dx$

**3**  $\displaystyle\int (2x + 3)\,(x^2 + 3x - 11)^4\,dx$

**4**  $\displaystyle\int 8x^3\sqrt{x^4 - 2}\,dx$

**5**  $\displaystyle\int \frac{2x}{x^2 - 6}\,dx$

**6**  $\displaystyle\int \cos x\,\sin^3 x\,\,dx$

# Answers

**1**    $\dfrac{1}{6}(x^3 + 8)^6 + c$

**2**    $\dfrac{1}{8}(x^2 + 10)^4 + c$

**3**    $\dfrac{1}{5}(x^2 + 3x - 11)^5 + c$

**4**    $\dfrac{4}{3}\sqrt{(x^4 - 2)^3}$

**5**    $\ln|x^2 - 6| + c$

**6**    $\dfrac{1}{4}\sin^4 x + c$