Outcome 3 – Integrating inverse trig functions with substitution

Worked Example:

Calculate $\int \frac{8}{\sqrt{49-4x^2}}\,dx$

$= 8\int \frac{1}{\sqrt{49-4x^2}}\,dx$

$= 8\int \frac{1}{\sqrt{7^2-u^2}}\,\frac{du}{2}$

$= 4\int \frac{1}{\sqrt{7^2-u^2}}\,du$

$= 4\sin^{-1}\left(\frac{u}{7}\right)+c$

$= 4\sin^{-1}\left(\frac{2x}{7}\right)+c$

1. Take the constant out
2. Use the formula sheet!

$\frac{1}{\sqrt{(7)^2-(2x)^2}}$

Let $u = 2x$

$\frac{du}{dx} = 2$

$du = 2\,dx$

$\frac{du}{2} = dx$

3. Make your substitution
4. Replace u with x term

Advanced Higher Formula sheet

$f(x)$	$\int f(x)\,dx$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right)+c$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$

Key Facts/Formulae:

If a function is multiplied by a constant, you can take the constant outside of the integral sign and deal with it at the end.

E.g. $\int \frac{1}{3x^2}dx = \frac{1}{3}\int \frac{1}{x^2}dx$

Questions…

Calculate each of the following integrals.

1. $\int \frac{21}{\sqrt{16-9x^2}}\,dx$
2. $\int \frac{6}{\sqrt{25-4x^2}}\,dx$
3. $\int \frac{24}{\sqrt{121-16x^2}}\,dx$
4. $\int \frac{16}{9+64x^2}\,dx$
5. $\int \frac{27}{4+81x^2}\,dx$
6. $\int \frac{1}{2(36+49x^2)}\,dx$

Answers

1 $7\sin^{-1}\left(\frac{3x}{4}\right)+c$

2 $3\sin^{-1}\left(\frac{2x}{5}\right)+c$

3 $6\sin^{-1}\left(\frac{4x}{11}\right)+c$

4 $\frac{2}{3}\tan^{-1}\left(\frac{8x}{3}\right)+c$

5 $\frac{3}{2}\tan^{-1}\left(\frac{9x}{2}\right)+c$

6 $\frac{1}{84}\tan^{-1}\left(\frac{7x}{6}\right)+c$