Outcome 2 – Integrating inverse trig functions with constants

Worked Example:

Calculate $\int \frac{6}{\sqrt{81-x^2}}\,dx$

1. Take the constant out

$$6\int \frac{1}{\sqrt{81-x^2}}\,dx = 6\sin^{-1}\left(\frac{x}{9}\right)+c$$

2. Use the formula sheet! $\frac{1}{\sqrt{(9)^2-(x)^2}}$

Advanced Higher Formula sheet

$f(x)$	$\int f(x)\,dx$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right)+c$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$

Key Facts/Formulae:

If a function is multiplied by a constant, you can take the constant outside of the integral sign and deal with it at the end.

E.g. $\int \frac{1}{3x^2}dx = \frac{1}{3}\int \frac{1}{x^2}dx$

Questions…

Calculate each of the following integrals.

1. $\int \frac{4}{\sqrt{9-x^2}}\,dx$
2. $\int \frac{9}{\sqrt{36-x^2}}\,dx$
3. $\int \frac{1}{3\sqrt{144-x^2}}\,dx$
4. $\int \frac{6}{64+x^2}\,dx$
5. $\int \frac{7}{49+x^2}\,dx$
6. $\int \frac{1}{2(16+x^2)}\,dx$

Answers

1. $4\sin^{-1}\left(\frac{x}{3}\right)+c$

2. $9\sin^{-1}\left(\frac{x}{6}\right)+c$

3. $\frac{1}{3}\sin^{-1}\left(\frac{x}{12}\right)+c$

4. $\frac{3}{4}\tan^{-1}\left(\frac{x}{8}\right)+c$

5. $\tan^{-1}\left(\frac{x}{7}\right)+c$

6. $\frac{1}{8}\tan^{-1}\left(\frac{x}{4}\right)+c$