Outcome 2

Parametric Differentiation – the 2nd derivative

Key Facts/Formulae:

Some graphs are drawn with different equations for the x and y coordinates.

These are called parametric equations.

The x and y coordinates are connected by an independent variable - often t.

The first derivative

$$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt}$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \left[\frac{d}{dt}\left(\frac{dy}{dx}\right)\right] \times \frac{dt}{dx}$$

The second derivative

$$\frac{d^2y}{dx^2} = \left[\frac{d}{dt}\left(\frac{dy}{dx}\right)\right] \times \frac{dt}{dx}$$

Worked Example:

A curve is given by the parametric equations $x = t^2 + 8$ and $y = 6t^3$.

Find $\frac{d^2y}{dx^2}$ in terms of t.

1. Differentiate both equations in terms of t

$$\frac{dx}{dt} = 2t \qquad \frac{dy}{dt} = 18t^2$$

2. Calculate the first derivative

$$\frac{dy}{dx} = \frac{18t^2}{2t} = 9t$$

3. Calculate the second derivative

$$\frac{d^2y}{dx^2} = 9 \times \frac{1}{2t} = \frac{9}{2t}$$

Questions…

Find $\frac{d^2y}{dx^2}$ for the curve defined by each pair of parametric equations;

1. $x = t^2 + 6$ and $y = 8t^3$
2. $x = t^3 - 8$ and $y = 2t^4$
3. $x = 3t + \frac{1}{3}t^3$ and $y = \frac{1}{4}t^4 - 4t$
4. $x = t^3$ and $y = \ln t$
5. $x = t^4$ and $y = \frac{1}{t}$
6. $x = t + \cos t$ and $y = t - \sin t$

Answers

1 $$\frac{d^2y}{dx^2} = \frac{6}{t}$$

2 $$\frac{d^2y}{dx^2} = -\frac{8}{9t^2}$$

3 $$\frac{d^2y}{dx^2} = \frac{t(t^3 + 9t + 8)}{(3 + t^2)^3}$$

4 $$\frac{d^2y}{dx^2} = -\frac{1}{3t^6}$$

5 $$\frac{d^2y}{dx^2} = -\frac{5}{16t^9}$$

6 $$\frac{d^2y}{dx^2} = \frac{\sin t + \cos t - 1}{(1 - \sin t)^3}$$