Outcome 3: Parametric Differentiation
Calculating instantaneous speed

Worked Example:

The position of a projectile on a coordinate axis system at time, t seconds, is given by;

$x = 30t - 5$ and $y = 40 - 3t^2$.

Calculate the speed of the projectile, in metres per second, after 2 seconds.

1. Differentiate both equations in terms of t

$$\frac{dx}{dt} = 30 \qquad \frac{dy}{dt} = -6t$$

2. Sub into the formula (using $t = 2$)

$$speed = \sqrt{\left(30\right)^2 + \left(-12\right)^2}$$

$$= \sqrt{900 + 144}$$

$$= 32 \cdot 3 \ m/sec$$

Key Facts/Formulae:

Some graphs are drawn with different equations for the x and y coordinates.

These are called parametric equations.

The x and y coordinates are connected by an independent variable - often t.

The first derivative

$$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt}$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \left[\frac{d}{dt}\left(\frac{dy}{dx}\right)\right] \times \frac{dt}{dx}$$

The second derivative

$$\frac{d^2y}{dx^2} = \left[\frac{d}{dt}\left(\frac{dy}{dx}\right)\right] \times \frac{dt}{dx}$$

(For $x = f(t)$ and $y = g(t)$)

Instantaneous speed

$$speed = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

Questions...

1. The position of a projectile on a coordinate axis system at time, t seconds, is given by;
$x = 40t - 9$ and $y = 50 - 4t^2$.

Calculate the speed of the projectile, in metres per second, after 5 seconds.

2. The position of a rocket on a coordinate axis system at time, t seconds, is given by;
$x = t^3$ and $y = -12t$.

Calculate the speed of the rocket, in metres per second, after 8 seconds.

3. The position of a tennis ball on a coordinate axis system at time, t seconds, is given by;
$x = t^2$ and $y = 2t^3$.

Calculate the speed of the tennis ball, in centimetres per second, after 2 seconds.

4. The position of a projectile on a coordinate axis system at time, t seconds, is given by;
$x = 2t + \frac{1}{2}t^2$ and $y = \frac{1}{3}t^3 - 3t$.

Calculate the speed of the projectile, in metres per second, after 4 seconds.

Answers

1 $56 \cdot 6\ m/sec$

2 $192 \cdot 4\ m/sec$

3 $24 \cdot 3\ cm/sec$

4 $14 \cdot 3\ m/sec$