Outcome 2

Implicit differentiation – with more than one 'y' term

Worked Example:

For $6y = x^4 - y^2$, use implicit differentiation to find $\frac{dy}{dx}$.

1. Differentiate each term "as normal"

$$6\frac{dy}{dx} = 4x^3 - 2y\frac{dy}{dx}$$

2. Make $\frac{dy}{dx}$ the subject of the formula

$$2y\frac{dy}{dx} + 6\frac{dy}{dx} = 4x^3$$

$$\frac{dy}{dx}(2y + 6) = 4x^3$$

$$\frac{dy}{dx} = \frac{4x^3}{2y + 6}$$

Key Facts/Formulae:

For some functions, it may not be possible (or practical) to state them with y as the subject.

To differentiate implicitly; $y = x^3$

1. Differentiate x terms "as normal". $\frac{dy}{dx} = 3x^2$
2. Differentiate y terms "as normal" and write $\frac{dy}{dx}$ at the end.
3. Make $\frac{dy}{dx}$ the subject of the formula.

Note: Your final answers will involve x and y.

Questions…

Differentiate each of the following with respect to x…

1. $4y = x^2 - y^3$
2. $5y = x^3 + y^4$
3. $3x + 2y = 4y^5$
4. $8y - y^3 = 10x^4$
5. $y - x^2 + y^7 = 0$
6. $5y^6 + 3x^8 - 2y^4 = 0$

Answers

1 $$\frac{dy}{dx} = \frac{2x}{3y^2 + 4}$$

2 $$\frac{dy}{dx} = \frac{3x^2}{5 - 4y^3}$$

3 $$\frac{dy}{dx} = \frac{3}{20y^4 - 2}$$

4 $$\frac{dy}{dx} = \frac{40x^3}{8 - 3y^2}$$

5 $$\frac{dy}{dx} = \frac{2x}{7y^6 + 1}$$

6 $$\frac{dy}{dx} = -\frac{24x^7}{2y^3(4 - 15y^2)}$$