Outcome 3

Implicit differentiation – using the product rule

Key Facts/Formulae:

For some functions, it may not be possible (or practical) to state them with y as the subject.

To differentiate implicitly; $y = x^3$

1. Differentiate x terms "as normal". $\frac{dy}{dx} = 3x^2$
2. Differentiate y terms "as normal" and write $\frac{dy}{dx}$ at the end.
3. Make $\frac{dy}{dx}$ the subject of the formula.

Note: Your final answers will involve x and y.

Worked Example:

Use the product rule!

For $4y = x^6 + xy^2$, use implicit differentiation to find $\frac{dy}{dx}$.

1. Differentiate each term "as normal"

$$4\frac{dy}{dx} = 6x^5 + x \times 2y\frac{dy}{dx} + y^2 \times 1$$

$$4\frac{dy}{dx} = 6x^5 + 2xy\frac{dy}{dx} + y^2$$

2. Make $\frac{dy}{dx}$ the subject of the formula

$$4\frac{dy}{dx} - 2xy\frac{dy}{dx} = 6x^5 + y^2$$

$$\frac{dy}{dx}(4 - 2xy) = 6x^5 + y^2$$

$$\frac{dy}{dx} = \frac{6x^5 + y^2}{4 - 2xy}$$

Questions…

Differentiate each of the following with respect to x…

1. $5y = x^4 + xy^3$
2. $7y = x^8 + xy^2$
3. $3y = x^2 + 2xy^4$
4. $4y - 6x = x^3y^5$
5. $9x + 2y^3 = 4x^2y^5$
6. $6x - x^4y^2 = 10y$

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Answers

1 $\frac{dy}{dx} = \frac{4x^3 + y^3}{5 - 3xy^2}$

2 $\frac{dy}{dx} = \frac{8x^7 + y^2}{7 - 2xy}$

3 $\frac{dy}{dx} = \frac{2x + 2y^4}{3 - 8xy^3}$

4 $\frac{dy}{dx} = \frac{3x^2y^5 + 6}{4 - 5x^3y^4}$

5 $\frac{dy}{dx} = \frac{8xy^5 - 9}{6y^5 - 20x^2y^4}$

6 $\frac{dy}{dx} = \frac{4x^3y^2 + 6}{2x^4y + 10}$

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