Outcome 1

Parametric Differentiation – the 1st derivative

Key Facts/Formulae:

Some graphs are drawn with different equations for the x and y coordinates.

These are called parametric equations.

The x and y coordinates are connected by an independent variable - often t.

The first derivative

$$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt}$$

Worked Example:

A curve is given by the parametric equations

$x = 8t$ and $y = 5 - \cos t$.

Find $\frac{dy}{dx}$ in terms of t.

1. Differentiate both equations in terms of t

$$\frac{dx}{dt} = 8 \qquad \frac{dy}{dt} = \sin t$$

2. Find $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{\sin t}{8} = \frac{1}{8}\sin t$$

Questions…

Find $\frac{dy}{dx}$ for the curve defined by each pair of parametric equations;

1. $x = 5t$ and $y = t^2$

2. $x = t^3$ and $y = -10t$

3. $x = 8t + 3$ and $y = 4t^2 - 7$

4. $x = 4t$ and $y = 2 - \sin t$

5. $x = \ln(4t + 1)$ and $y = t^2 + 5$

6. $x = \ln(1 + t)$ and $y = \ln(1 + t^2)$

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Answers

1 $$\frac{dy}{dx} = \frac{2t}{5}$$

2 $$\frac{dy}{dx} = -\frac{10}{3t^2}$$

3 $$\frac{dy}{dx} = t$$

4 $$\frac{dy}{dx} = -\frac{1}{4}\cos t$$

5 $$\frac{dy}{dx} = 2t^2 + \frac{1}{2}t$$

6 $$\frac{dy}{dx} = \frac{2t(1+t)}{1+t^2}$$

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