Outcome 2

Derivatives of inverse trig functions – $\cos^{-1}x$

Key Facts/Formulae:

$f(x)$	$f'(x)$
$\sin^{-1}x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1}x$	$-\frac{1}{\sqrt{1-x^2}}$

There are formulae that enable us to differentiate inverse trigonometric functions.

f'(outside) × f'(inside)

Given on formula sheet! ✓

Worked Example:

Given $f(x) = \cos^{-1} 5x$, find $f'(x)$.

1. Sub into the formula.

$$f'(x) = -\frac{1}{\sqrt{1-(5x)^2}} \times 5 = -\frac{5}{\sqrt{1-25x^2}}$$

2. Multiply by the derivative of the function within the function and expand the bracket.

Questions…

Differentiate each of the following with respect to x…

1. $f(x) = \cos^{-1} 2x$
2. $y = \cos^{-1} 6x$
3. $f(x) = \cos^{-1} 8x^2$
4. $y = \cos^{-1} x^4$
5. $f(x) = x \cos^{-1} 7x$
6. $y = 10x^2 \cos^{-1} 9x$

Answers

1 $$f'(x) = -\frac{2}{\sqrt{1-4x^2}}$$

2 $$\frac{dy}{dx} = -\frac{6}{\sqrt{1-36x^2}}$$

3 $$f'(x) = -\frac{16x}{\sqrt{1-64x^4}}$$

4 $$\frac{dy}{dx} = -\frac{4x^3}{\sqrt{1-x^8}}$$

5 $$f'(x) = -\frac{7x}{\sqrt{1-49x^2}} + \cos^{-1} 9x$$

6 $$\frac{dy}{dx} = -\frac{90x^2}{\sqrt{1-81x^2}} + 20x\cos^{-1} 9x$$