Outcome 3

Derivatives of inverse trig functions – tan^{-1}x

Worked Example:

Given $f(x) = \tan^{-1} 7x$, find $f'(x)$.

1. Sub into the formula.

$$f'(x) = \frac{1}{1+(7x)^2} \times 7 = \frac{7}{1+49x^2}$$

2. Multiply by the derivative of the function within the function and expand the bracket.

Key Facts/Formulae:

$f(x)$	$f'(x)$
$\sin^{-1} x$	$\dfrac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\dfrac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\dfrac{1}{1+x^2}$

There are formulae that enable us to differentiate inverse trigonometric functions.

f'(outside) × f'(inside)

Given on formula sheet! ✔

Questions...

Differentiate each of the following with respect to x...

1 $f(x) = \tan^{-1} 3x$

2 $y = \tan^{-1} 8x$

3 $f(x) = \tan^{-1} 4x^2$

4 $y = \tan^{-1} x^5$

5 $f(x) = \sin x \tan^{-1} 5x$

6 $y = 6x^2 \tan^{-1} 10x$

Answers

1 $f'(x) = \dfrac{3}{1 + 9x^2}$

2 $\dfrac{dy}{dx} = \dfrac{8}{1 + 64x^2}$

3 $f'(x) = \dfrac{8x}{1 + 16x^4}$

4 $\dfrac{dy}{dx} = \dfrac{5x^4}{1 + x^{10}}$

5 $f'(x) = \dfrac{5 \sin x}{1 + 25x^2} + \cos x \tan^{-1} 5x$

6 $\dfrac{dy}{dx} = \dfrac{60x^2}{1 + 100x^2} + 12x \tan^{-1} 10x$