

Outcome 4 - The Chain Rule, Product Rule and Quotient Rule with further trigonometric functions

Worked Example:

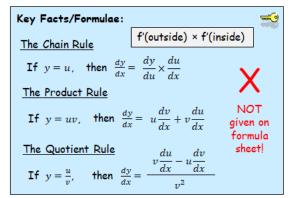
Show that the derivative of $\sec x$ is $\sec x \tan x$.

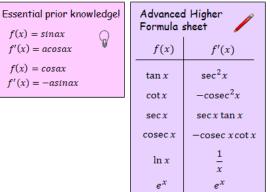
1. Define function and get in a differentiable form.

Let
$$y = \sec x = \frac{1}{\cos x} = (\cos x)^{-1}$$

2. Choose strategy and differentiate. (Chain Rule)

$$\frac{dy}{dx} = -(\cos x)^{-2} \times (-\sin x) = \frac{\sin x}{\cos^2 x}$$
$$= \frac{1}{\cos x} \times \frac{\sin x}{\cos x} = \sec x \tan x \quad \text{as required.}$$





Questions...

Prove each of the following;

- \triangle Show that the derivative of $\tan x$ is $\sec^2 x$.
- \clubsuit Show that the derivative of $\cot x$ is $-\csc^2 x$.
- Show that the derivative of $\sec x \cos^2 x$ is $-\sin x$.
- \clubsuit Show that the derivative of $\sec^2 x \cos^2 x$ is 0.
- 5 Show that the derivative of $\tan^2 x + 1$ is $2 \tan x \sec^2 x$.
- Show that the derivative of $\csc x + \cot x$ is $-\frac{1+\cos x}{\sin^2 x}$.

Answers

$$y = \tan x = \frac{\sin x}{\cos x}$$
 Quotient rule!
$$\frac{dy}{dx} = \frac{\cos^2 x - \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

$$y = \cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$$
 Quotient rule!

$$\frac{dy}{dx} = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = -\frac{\cos^2 x + \sin^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x} = -\csc^2 x$$

$$y = \sec x \cos^2 x = \frac{1}{\cos x} \times \cos^2 x = \frac{\cos^2 x}{\cos x} = \cos x$$

$$\frac{dy}{dx} = -\sin x$$
Basic differentiation!

$$y = \sec^2 x \cos^2 x = \frac{1}{\cos^2 x} \times \cos^2 x = \frac{\cos^2 x}{\cos^2 x} = 1$$

$$\frac{dy}{dx} = 0$$
Basic differentiation!

$$y = \tan^2 x + 1 = \frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{\sin^2 x + \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = (\cos x)^{-2}$$

$$\frac{dy}{dx} = -2(\cos x)^{-3} \times (-\sin x) = 2 \times \frac{\sin x}{\cos x} \times \frac{1}{\cos^2 x} = 2 \tan x \sec^2 x \quad \text{Chain rule!}$$

$$y = \csc x + \cot x = \frac{1}{\sin x} + \frac{1}{\tan x} = \frac{1}{\sin x} + \frac{\cos x}{\sin x} = \frac{1 + \cos x}{\sin x}$$
 Quotient rule!
$$\frac{dy}{dx} = \frac{-\sin^2 x - \cos x (1 + \cos x)}{\sin^2 x} = \frac{-\sin^2 x - \cos x - \cos^2 x}{\sin^2 x} = -\frac{\cos^2 x + \sin^2 x + \cos x}{\sin^2 x} = -\frac{1 + \cos x}{\sin^2 x}$$