

#### Outcome 3 - The Quotient Rule with "Advanced Higher" derivatives

#### Worked Example:

Differentiate  $y = \frac{\cos 6x}{e^{3x}}$  and simplify your answer.

#### 1. Define the functions.

Let 
$$y = \frac{u}{v}$$
 where  $u = \cos 6x$  and  $v = e^{3x}$ 

#### 2. Differentiate both functions.

$$\frac{du}{dx} = -\sin 6x \times 6 = -6\sin 6x \qquad \frac{dv}{dx} = e^{3x} \times 3 = 3e^{3x}$$

### 3. Find $\frac{dy}{dx}$ .

$$\frac{dy}{dx} = \frac{-6\sin 6x \, e^{3x} - 3\cos 6x \, e^{3x}}{e^{3x} \times e^{3x}} = \frac{-6\sin 6x - 3\cos 6x}{e^{3x}}$$

### Key Facts/Formulae:

f(x) = sinaxf'(x) = a cos a x

 $f(x) = \cos ax$ 

f(x)

 $\ln f(x)$ 

 $e^{f(x)}$ 

f'(x) = -asinax

Formulae NOT on sheet!

f'(x)

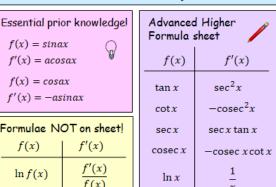
f'(x)

 $\overline{f(x)}$ 

 $f'(x)e^{f(x)}$ 

The quotient rule enables us to differentiate a rational function where both the numerator and denominator are functions

we can differentiate easily.   
E.g. If 
$$y = \frac{u}{v}$$
, then  $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ 



## Questions...

Differentiate each of the following with respect to x, leaving your answers in their simplest form.

$$4 \qquad y = \frac{e^x}{3x+1}$$

$$3 \qquad y = \frac{x^2 + 1}{e^{4x}}$$

$$4 \qquad y = \frac{\cos^3 x}{e^{5x}}$$

$$y = \frac{\ln 5x}{\sin^2 x}$$

# Answers

$$\frac{dy}{dx} = \frac{e^x(3x-2)}{(3x+1)^2}$$

$$\frac{dy}{dx} = \frac{2e^{2x}(4x-5)}{(4x-3)^2}$$

$$\frac{dy}{dx} = \frac{2(x - 2x^2 - 2)}{e^{4x}}$$

$$\frac{dy}{dx} = -\frac{\cos^2 x (3\sin x + 5\cos x)}{e^{5x}}$$

$$\frac{dy}{dx} = \frac{\cos x + x \sin x \ln x}{\cos^2 x}$$

$$\frac{dy}{dx} = \frac{\sin x - 2x \cos x \ln 5x}{x \sin^3 x}$$