Outcome 3 – The Quotient Rule with "Advanced Higher" derivatives

Worked Example:

Differentiate $y = \frac{\cos 6x}{e^{3x}}$ and simplify your answer.

1. Define the functions.

Let $y = \frac{u}{v}$ where $u = \cos 6x$ and $v = e^{3x}$

2. Differentiate both functions.

$\frac{du}{dx} = -\sin 6x \times 6 = -6\sin 6x$ $\frac{dv}{dx} = e^{3x} \times 3 = 3e^{3x}$

3. Find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{-6\sin 6x\, e^{3x} - 3\cos 6x\, e^{3x}}{e^{3x} \times e^{3x}} = \frac{-6\sin 6x - 3\cos 6x}{e^{3x}}$$

Key Facts/Formulae:

The quotient rule enables us to differentiate a rational function where both the numerator and denominator are functions we can differentiate easily.

E.g. If $y = \frac{u}{v}$, then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

NOT given on formula sheet!

Essential prior knowledge!

$f(x) = \sin ax$
$f'(x) = a\cos ax$

$f(x) = \cos ax$
$f'(x) = -a\sin ax$

Formulae NOT on sheet!

$f(x)$	$f'(x)$
$\ln f(x)$	$\frac{f'(x)}{f(x)}$
$e^{f(x)}$	$f'(x)e^{f(x)}$

Advanced Higher Formula sheet

$f(x)$	$f'(x)$
$\tan x$	$\sec^2 x$
$\cot x$	$-\text{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\text{cosec}\, x$	$-\text{cosec}\, x \cot x$
$\ln x$	$\frac{1}{x}$
e^x	e^x

Questions...

Differentiate each of the following with respect to x, leaving your answers in their simplest form.

1. $y = \frac{e^x}{3x + 1}$

2. $y = \frac{e^{2x}}{4x - 3}$

3. $y = \frac{x^2 + 1}{e^{4x}}$

4. $y = \frac{\cos^3 x}{e^{5x}}$

5. $y = \frac{\ln x}{\cos x}$

6. $y = \frac{\ln 5x}{\sin^2 x}$

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Answers

1 $$\frac{dy}{dx} = \frac{e^{x}(3x-2)}{(3x+1)^2}$$

2 $$\frac{dy}{dx} = \frac{2e^{2x}(4x-5)}{(4x-3)^2}$$

3 $$\frac{dy}{dx} = \frac{2(x-2x^2-2)}{e^{4x}}$$

4 $$\frac{dy}{dx} = -\frac{\cos^2 x(3\sin x + 5\cos x)}{e^{5x}}$$

5 $$\frac{dy}{dx} = \frac{\cos x + x\sin x \ln x}{\cos^2 x}$$

6 $$\frac{dy}{dx} = \frac{\sin x - 2x\cos x \ln 5x}{x\sin^3 x}$$

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