Outcome 2 – The Quotient Rule with the Chain Rule

Worked Example:

Differentiate $y = \frac{(3x-4)^5}{\sin^2 x}$ and simplify your answer.

1. Define the functions.

Let $y = \frac{u}{v}$ where $u = (3x - 4)^5$ and $v = \sin^2 x$ $(\sin x)^2$

2. Differentiate both functions.

$$\frac{du}{dx} = 5(3x-4)^4 \times 3 \qquad \frac{dv}{dx} = 2(\sin x) \times \cos x$$

3. Find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{15\sin^2 x\,(3x-4)^4 - 2\sin x\cos x\,(3x-4)^5}{\sin^4 x} = \frac{15\sin x\,(3x-4)^4 - 2\cos x\,(3x-4)^5}{\sin^3 x}$$

Key Facts/Formulae:

The quotient rule enables us to differentiate a rational function where both the numerator and denominator are functions we can differentiate easily.

E.g. If $y = \frac{u}{v}$, then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

NOT given on formula sheet!

Essential prior knowledge!

$f(x) = \sin ax \qquad f'(x) = a\cos ax$

$f(x) = \cos ax \qquad f'(x) = -a\sin ax$

Key Facts/Formulae:

f'(outside) × f'(inside)

E.g. If $y = u$, then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

Questions…

Differentiate each of the following with respect to x, leaving your answers in their simplest form.

1. $y = \frac{6x-1}{(2x+1)^3}$

2. $y = \frac{(x+3)^2}{5x+2}$

3. $y = \frac{(x+1)^2}{(x-1)^3}$

4. $y = \frac{\sin x}{(4x-1)^5}$

5. $y = \frac{(x-2)^6}{\cos^2 x}$

6. $y = \frac{(2x-9)^4}{\sin^3 x}$

Answers

1 $$\frac{dy}{dx} = \frac{6(4-x)}{(2x+1)^4}$$

2 $$\frac{dy}{dx} = \frac{(x+3)(5x-11)}{(5x+2)^2}$$

3 $$\frac{dy}{dx} = -\frac{(x+1)(x-5)}{(x-1)^4}$$

4 $$\frac{dy}{dx} = \frac{\cos x\,(4x-1) - 20\sin x}{(4x-1)^6}$$

5 $$\frac{dy}{dx} = \frac{2(x-2)^5[3\cos x + \sin x(x-2)]}{\cos^3 x}$$

6 $$\frac{dy}{dx} = \frac{(2x-9)^3[8\sin x - 3\cos x(2x-9)]}{\sin^4 x}$$