Outcome 1 – The Quotient Rule with basic functions

Worked Example:

Differentiate $y = \frac{x^4}{3x+1}$, giving your answer in its simplest form.

1. Define the functions.

Let $y = \frac{u}{v}$ where $u = x^4$ and $v = 3x + 1$

2. Differentiate both functions.

$$\frac{du}{dx} = 4x^3 \qquad \frac{dv}{dx} = 3$$

3. Find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{4x^3(3x+1) - 3x^4}{(3x+1)^2} = \frac{12x^4 + 4x^3 - 3x^4}{(3x+1)^2} = \frac{9x^4 + 4x^3}{(3x+1)^2} = \frac{x^3(9x+4)}{(3x+1)^2}$$

Key Facts/Formulae:

The quotient rule enables us to differentiate a rational function where both the numerator and denominator are functions we can differentiate easily.

E.g. If $y = \frac{u}{v}$, then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

NOT given on formula sheet!

Essential prior knowledge!

$f(x) = \sin ax \quad f'(x) = a\cos ax$

$f(x) = \cos ax \quad f'(x) = -a\sin ax$

Questions…

Differentiate each of the following with respect to x, leaving your answers in their simplest form.

1. $y = \frac{x}{5x+3}$

2. $y = \frac{x^2}{4x-1}$

3. $y = \frac{x^3+1}{x^2+1}$

4. $y = \frac{2x-3}{x^2+3}$

5. $y = \frac{x^2+2}{x^3-4}$

6. $y = \frac{6x}{\sin x}$

Answers

1 $$\frac{dy}{dx} = \frac{3}{(5x+3)^2}$$

2 $$\frac{dy}{dx} = \frac{2x(2x-1)}{(4x-1)^2}$$

3 $$\frac{dy}{dx} = \frac{x(x^3+3x-2)}{(x^2+1)^2}$$

4 $$\frac{dy}{dx} = \frac{2(3+3x-x^2)}{(x^2+3)^2}$$

5 $$\frac{dy}{dx} = -\frac{x(x^3+6x+8)}{(x^3-4)^2}$$

6 $$\frac{dy}{dx} = \frac{6(\sin x - x\cos x)}{\sin^2 x}$$