Outcome 3 – The Chain Rule for Advanced Higher derivatives

Worked Example 1:

Differentiate $y = \ln 7x$

Let $y = \ln u$ where $u = 7x$

$$\frac{dy}{du} = \frac{1}{u} \qquad \frac{du}{dx} = 7$$

$$\frac{dy}{dx} = \frac{7}{u} = \frac{7}{7x}$$

Worked Example 2:

Differentiate $y = e^{\cos x}$

Let $y = e^u$ where $u = \cos x$

$$\frac{dy}{du} = e^u \qquad \frac{du}{dx} = -\sin x$$

$$\frac{dy}{dx} = -\sin x\, e^u = -\sin x\, e^{\cos x}$$

Key Facts/Formulae:

The chain rule enables us to differentiate a function within a function. f'(outside) × f'(inside)

E.g. If $y = u$, then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

Essential prior knowledge!

$f(x) = \sin ax \quad f'(x) = a\cos ax$

$f(x) = \cos ax \quad f'(x) = -a\sin ax$

Advanced Higher Formula sheet

$f(x)$	$f'(x)$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\ln x$	$\frac{1}{x}$
e^x	e^x

Formulae NOT on sheet!

$f(x)$	$f'(x)$
$\ln f(x)$	$\frac{f'(x)}{f(x)}$
$e^{f(x)}$	$f'(x)e^{f(x)}$

Questions…

Differentiate each of the following with respect to x.

1. $y = \ln 3x$
2. $y = \ln(5x - 2)$
3. $y = \ln(x^2 - x - 6)$
4. $y = e^{6x+1}$
5. $y = e^{x^3}$
6. $y = e^{2\sin 3x}$

Answers

1 $$\frac{dy}{dx} = \frac{3}{3x}$$

2 $$\frac{dy}{dx} = \frac{2x - 1}{x^2 - x - 6}$$

3 $$\frac{dy}{dx} = -\frac{12}{(4x + 7)^4}$$

4 $$\frac{dy}{dx} = 6e^{6x+1}$$

5 $$\frac{dy}{dx} = 3x^2 e^{x^3}$$

6 $$\frac{dy}{dx} = 6cos3xe^{2sin3x}$$