Outcome 2 – The Chain Rule for more than two functions

Worked Example:

Key Facts/Formulae:
The chain rule can be extended to enable us to differentiate a function within a function within a function.

E.g. If $y = f(u)$, where $u = g(t)$, where $t = h(x)$,

then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dt} \times \frac{dt}{dx}$

Essential prior knowledge!

$f(x) = sinax \quad f'(x) = acosax$

$f(x) = cosax \quad f'(x) = -asinax$

Differentiate $y = cos^2 4x = (cos4x)^2$

1. Define the functions.

Let $y = u^2$ where $u = cos4x$ and let $u = cost$ where $t = 4x$

2. Differentiate the functions y, u and t.

$$\frac{dy}{du} = 2u \qquad \frac{du}{dt} = -sint \qquad \frac{dt}{dx} = 4$$

3. Find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = 2u \times (-sint) \times 4 = -8\, sint\, u$$

4. Replace t and u with the functions of x.

$$\frac{dy}{dx} = -8\, sin4x\, cos4x$$

Questions…

Differentiate each of the following with respect to x.

1. $y = cos^4 2x$

2. $y = sin^2(cosx)$

3. $y = (x + cos3x)^2$

4. $y = sin^3(5x + 1)$

5. $y = \dfrac{1}{\cos^2(9x + 4)}$

6. $y = \dfrac{1}{\cos(\sin x)}$

Answers

1 $$\frac{dy}{dx} = 8cos^3 2x\ sin2x$$

2 $$\frac{dy}{dx} = -2sinx\sin(cosx)\cos(cosx)$$

3 $$\frac{dy}{dx} = 2(x + cos3x)(1 - 3sin3x)$$

4 $$\frac{dy}{dx} = 15\sin^2(5x + 1)\cos(5x + 1)$$

5 $$\frac{dy}{dx} = \frac{18\sin(9x + 4)}{\cos^3(9x + 4)}$$

6 $$\frac{dy}{dx} = \frac{cosx\sin(sinx)}{cos^2(sinx)}$$