Outcome 1 – The Chain Rule for two functions

Key Facts/Formulae:

The chain rule enables us to differentiate a function within a function.

f'(outside) × f'(inside)

E.g. If $y = u$, then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

Essential prior knowledge!

$f(x) = \sin ax$ $f'(x) = a\cos ax$

$f(x) = \cos ax$ $f'(x) = -a\sin ax$

Worked Example:

Differentiate $y = (5x - 2)^7$

1. Define the functions.

Let $y = u^7$ where $u = 5x - 2$

2. Differentiate both functions.

$\frac{dy}{du} = 7u^6$ $\frac{du}{dx} = 5$

3. Find $\frac{dy}{dx}$.

$\frac{dy}{dx} = 7u^6 \times 5 = 35u^6$

4. Replace u with function of x.

$\frac{dy}{dx} = 35(5x - 2)^6$

Questions…

Differentiate each of the following with respect to x.

1. $y = (3x + 1)^2$

2. $y = (9x - 5)^8$

3. $y = \frac{1}{(4x + 7)^3}$

4. $y = \sin 8x$

5. $y = \cos x^3$

6. $y = \frac{1}{\cos x}$

Answers

1 $\frac{dy}{dx} = 6(3x + 1)$

2 $\frac{dy}{dx} = 72(9x - 5)^7$

3 $\frac{dy}{dx} = -\frac{12}{(4x + 7)^4}$

4 $\frac{dy}{dx} = 8\cos 8x$

5 $\frac{dy}{dx} = -3x^2 \sin x^3$

6 $\frac{dy}{dx} = \frac{\sin x}{cos^2 x}$