Outcome 2 – The Product Rule with bronze/silver Chain Rule

Key Facts/Formulae:

The chain rule can be extended to enable us to differentiate a function within a function within a function.

E.g. If $y = f(u)$, where $u = g(t)$, where $t = h(x)$,

then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dt} \times \frac{dt}{dx}$

Essential prior knowledge!

$f(x) = sinax \quad f'(x) = acosax$

$f(x) = cosax \quad f'(x) = -asinax$

Worked Example:

Differentiate $y = 4xcos2x$

1. Define the functions.

Let $y = uv$ where $u = 4x$ and $v = cos2x$

2. Differentiate both functions.

$$\frac{du}{dx} = 4 \qquad \frac{dv}{dx} = -2sin2x$$

3. Find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = -8x\,sin2x + 4\,cos2x$$

Questions…

Differentiate each of the following with respect to x.

1. $y = 3x \sin 4x$

2. $y = 5x \cos 6x$

3. $y = x^2 \sin 2x$

4. $y = 10x^3 \cos 3x$

5. $y = 2x^5 \sin 2x$

6. $y = 3x^4 \cos 8x$

Answers

1 $$\frac{dy}{dx} = 12x\cos 4x + 3\sin 4x$$

2 $$\frac{dy}{dx} = -30x\sin 6x + 5\cos 6x$$

3 $$\frac{dy}{dx} = 2x^2\cos 2x + 2x\sin 2x$$

4 $$\frac{dy}{dx} = -30x^3\sin 3x + 30x^2\cos 3x$$

5 $$\frac{dy}{dx} = 4x^5\cos 2x + 10x^4\sin 2x$$

6 $$\frac{dy}{dx} = -24x^4\sin 8x + 12x^3\sin 8x$$