Outcome 3 – The Product Rule with gold level Chain Rule

Worked Example:

Differentiate $y = sin^2 x cos^3 x$

Key Facts/Formulae:

The product rule enables us to differentiate a function consisting of two other functions that are multiplied together.

E.g. If $y = uv$, then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

Essential prior knowledge!

$f(x) = sinax \quad f'(x) = acosax$

$f(x) = cosax \quad f'(x) = -asinax$

1. Define the functions.

$(sinx)^2 \qquad (cosx)^3$

Let $y = uv$ where $u = sin^2 x$ and $v = cos^3 x$

2. Differentiate both functions.

$$\frac{du}{dx} = 2(sinx) \times cosx \qquad \frac{dv}{dx} = 3(cosx)^2 \times (-sinx)$$

$$= 2\,sinx\,cosx \qquad = -3\,sinx\,cos^2 x$$

3. Find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = -3\,sin^3 x\,cos^2 x + 2\,sinx\,cos^4 x$$

Questions…

Differentiate each of the following with respect to x.

1. $y = \sin^4 x \cos^2 x$
2. $y = \sin^2 x \cos^4 x$
3. $y = 2\sin^3 x \cos^2 x$
4. $y = 3\sin^2 x \cos^4 x$
5. $y = 10\sin^4 x \cos^3 x$
6. $y = \frac{1}{2}\sin^2 x \cos^2 x$

Answers

1 $\frac{dy}{dx} = -2\sin^5 x \cos x + 4\sin^3 x \cos^3 x$

2 $\frac{dy}{dx} = -4\sin^3 x \cos^3 x + 2\sin x \cos^5 x$

3 $\frac{dy}{dx} = -4\sin^4 x \cos x + 6\sin^2 x \cos^3 x$

4 $\frac{dy}{dx} = -12\sin^3 x \cos^3 x + 6\sin x \cos^5 x$

5 $\frac{dy}{dx} = -30\sin^5 x \cos^2 x + 40\sin^3 x \cos^4 x$

6 $\frac{dy}{dx} = -\sin^3 x \cos x + \sin x \cos^3 x$