Outcome 4 – The Product Rule with Advanced Higher derivatives

Key Facts/Formulae:

The product rule enables us to differentiate a function consisting of two other functions that are multiplied together.

E.g. If $y = uv$, then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

Essential prior knowledge!

$f(x) = sinax$ $f'(x) = acosax$

$f(x) = cosax$ $f'(x) = -asinax$

Worked Example:

Differentiate $y = e^{3x}cos^2x$

1. Define the functions.

Let $y = uv$ where $u = e^{3x}$ and $v = cos^2x$

$(cosx)^2$

2. Differentiate both functions.

$\frac{du}{dx} = e^{3x} \times 3 = 3e^{3x}$ $\frac{dv}{dx} = 2(cosx) \times (-sinx) = -2sinxcosx$

3. Find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = e^{3x} \times -2sinxcosx + cos^2x \times 3e^{3x}$$
$$= -2e^{3x}sinxcosx + 3e^{3x}cos^2x$$

Advanced Higher Formula sheet

$f(x)$	$f'(x)$
$\tan x$	$\sec^2 x$
$\cot x$	$-\text{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\text{cosec}\, x$	$-\text{cosec}\, x \cot x$
$\ln x$	$\frac{1}{x}$
e^x	e^x

Formulae NOT on sheet!

$f(x)$	$f'(x)$
$\ln f(x)$	$\frac{f'(x)}{f(x)}$
$e^{f(x)}$	$f'(x)e^{f(x)}$

Questions…

Differentiate each of the following with respect to x.

1. $y = x^2e^x$
2. $y = e^xsinx$
3. $y = e^{2x}cos^4x$
4. $y = 3x^2 \ln x$
5. $y = \ln(5x + 2)\sin^2 x$
6. $y = \ln(4x - 1)\tan x$

© Cdmasterworks Ltd 2012

Answers

1 $\frac{dy}{dx} = x^2e^x + 2xe^x$

2 $\frac{dy}{dx} = e^x \cos x + e^x \sin x$

3 $\frac{dy}{dx} = -4e^{2x} \sin x \cos^3 x + 2e^{2x}\cos^4 x$

4 $\frac{dy}{dx} = 3x + 6x\ln x$

5 $\frac{dy}{dx} = \sin 2x \ln(5x + 2) + \frac{5 \sin^2 x}{5x + 2}$

6 $\frac{dy}{dx} = \sec^2 x \ln(4x - 1) + \frac{4 \tan x}{4x - 1}$

© Cdmasterworks Ltd 2012