Outcome 4 – Improper Fractions

Key Facts/Formulae:

We can only use the partial fraction technique on proper fractions. If the fraction is an improper fraction, then we must use algebraic division first.

Worked Example:

Express $\frac{3x^2+3x+2}{x^2+2x-3}$ as the sum of a polynomial and partial fractions.

1. Do algebraic division to get a proper fraction.

$$\begin{array}{r|l} & 3 \\ \hline x^2+2x-3 & 3x^2+3x+2 \\ & 3x^2+6x-9 \\ \hline & -3x+11 \end{array}$$

$$3+\frac{-3x+11}{x^2+2x-3}$$

2. Use partial fraction technique on the proper fraction.

Let $\frac{-3x+11}{(x+3)(x-1)} = \frac{A}{(x+3)} + \frac{B}{(x-1)}$

Multiply all through by $(x+3)(x-1)$

$$-3x+11 = A(x-1)+B(x+3)$$

Sub in $x=-3$

$$20 = -4A \quad A = -5$$

Sub in $x=1$

$$8 = 4B \quad B = 2$$

3. Write final answer.

$$\frac{3x^2+3x+2}{x^2+2x-3} = 3+\frac{2}{(x-1)}-\frac{5}{(x+3)}$$

Questions…

Express each of the following as the sum of a polynomial and partial fractions.

1. $\frac{5x^2-12x-47}{x^2-3x-4}$

2. $\frac{7x^2+59x+116}{x^2+8x+15}$

3. $\frac{2x^2-6x-15}{x^2-5x-6}$

4. $\frac{x^2-6x-73}{x^2+x-20}$

5. $\frac{6x^2+9x-22}{x^2+x-6}$

6. $\frac{x^3-3x}{x^2-x-2}$

Answers

1 $5+\frac{6}{(x+1)}-\frac{3}{(x-4)}$

2 $7+\frac{1}{(x+3)}+\frac{2}{(x+5)}$

3 $2+\frac{3}{(x-6)}+\frac{1}{(x+1)}$

4 $1+\frac{2}{(x+5)}-\frac{9}{(x-4)}$

5 $6+\frac{4}{(x-2)}-\frac{1}{(x+3)}$

6 $x+1+\frac{2}{3(x-2)}-\frac{2}{3(x+1)}$