Outcome 3 – Irreducible Quadratic Factors

Worked Example:

Express $\frac{5x^2+6x+9}{(x+1)(x^2+2x+5)}$ in partial fractions.

1. Confirm that the quadratic is irreducible.

x^2+2x+5 $\quad b^2-4ac = 4-4(5)$
$= 4-20 = -16$

2. Begin process with the general formula $\quad a=1 \quad b=2 \quad c=5$

Let $\frac{5x^2+6x+9}{(x+1)(x^2+2x+5)} = \frac{A}{(x+1)} + \frac{Bx+C}{(x^2+2x+5)}$

Since $b^2-4ac<0$, there are no real roots

3. Multiply all through by the LCM of the denominators

$(x+1)(x^2+2x+5)$

$5x^2+6x+9 = A(x^2+2x+5)+(Bx+C)(x+1)$

4. Sub in an 'x' value (preferably a root)

$x=-1 \qquad 8=4A \qquad A=2$

5. Sub in x = 0 to eliminate B

$x=0 \qquad 9=5A+C \qquad 9=10+C \qquad C=-1$

6. Sub in another 'x' value (preferably the most straight forward one you haven't used yet!)

$x=1 \qquad 20=8A+2B+2C \qquad 20=16+2B-2 \qquad 2B=6 \qquad B=3$

7. Answer the question!

$$\frac{5x^2+6x+9}{(x+1)(x^2+2x+5)} = \frac{2}{(x+1)} + \frac{3x-1}{(x^2+2x+5)}$$

Key Facts/Formulae:

If the denominator contains a quadratic factor that cannot be factorised then the numerator of the partial fraction corresponding to this factor is of the form Bx + C.

E.g. $\frac{5x^2+6x+9}{(x+1)(x^2+2x+5)} = \frac{A}{(x+1)} + \frac{Bx+C}{(x^2+2x+5)}$

Questions…

Express each of the following in partial fractions.

1. $\frac{4x^2+16x+22}{(x+2)(x^2+3x+4)}$

2. $\frac{4x^2+31x+27}{(x+3)(x^2+5x+1)}$

3. $\frac{16x^2+62x+6}{(x+4)(x^2+4x+2)}$

4. $\frac{14x^2+53x+103}{(x-1)(x^2+6x+10)}$

5. $\frac{14x^2+13x+7}{(x+1)(x^2+x+1)}$

6. $\frac{-4x^2+16x-18}{(x-5)(x^2+x+8)}$

Answers

1. $\dfrac{3}{(x+2)} + \dfrac{x+5}{(x^2+3x+4)}$

2. $\dfrac{6}{(x+3)} + \dfrac{7-2x}{(x^2+5x+1)}$

3. $\dfrac{7}{(x+4)} + \dfrac{9x-2}{(x^2+4x+2)}$

4. $\dfrac{10}{(x-1)} + \dfrac{4x-3}{(x^2+6x+10)}$

5. $\dfrac{8}{(x+1)} + \dfrac{6x-1}{(x^2+x+1)}$

6. $-\dfrac{1}{(x-5)} + \dfrac{2-3x}{(x^2+x+8)}$