Outcome 2 – Repeated Linear Factors

Key Facts/Formulae:

If the denominator contains the same bracket more than once then more than one partial fraction must be included for this factor.

E.g. $\frac{3x^2-3x-9}{(x+2)(x-1)^2} = \frac{A}{(x+2)} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}$

Worked Example:

Express $\frac{3x^2-3x-9}{(x+2)(x-1)^2}$ in partial fractions.

1. Begin process with the general formula

Let $\frac{3x^2-3x-9}{(x+2)(x-1)^2} = \frac{A}{(x+2)} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}$

2. Multiply all through by the LCM of the denominators

$(x+2)(x-1)^2$

$3x^2 - 3x - 9 = A(x-1)^2 + B(x+2)(x-1) + C(x+2)$

3. Sub in an 'x' value (preferably a root)

$x = 1$ $\quad -9 = 3C \quad C = -3$

4. Sub in another 'x' value (preferably a root)

$x = -2$ $\quad 9 = 9A \quad A = 1$

5. Sub in another 'x' value (preferably the most straight forward one you haven't used yet!)

$x = 0$ $\quad -9 = A - 2B + 2C \quad 2B = 4$

$-9 = 1 - 2B - 6 \quad B = 2$

6. Answer the question!

$\frac{3x^2-3x-9}{(x+2)(x-1)^2} = \frac{1}{(x+2)} + \frac{2}{(x-1)} - \frac{3}{(x-1)^2}$

Questions…

Express each of the following in partial fractions.

1. $\frac{9x^2+4x+1}{(x-2)(x+1)^2}$

2. $\frac{34-4x-3x^2}{(x-1)(x+2)^2}$

3. $\frac{11x^2+47x+51}{(x+3)(x+2)^2}$

4. $\frac{3x^2-27x+33}{(x+1)(x-2)^2}$

5. $\frac{99+50x-x^2}{(x-7)(x+3)^2}$

6. $\frac{9x^2-28x+40}{x^2(x-5)}$

Answers

1. $\frac{5}{(x-2)}+\frac{4}{(x+1)}-\frac{2}{(x+1)^2}$

2. $\frac{3}{(x-1)}-\frac{6}{(x+2)}-\frac{10}{(x+2)^2}$

3. $\frac{9}{(x+3)}+\frac{2}{(x+2)}+\frac{1}{(x+2)^2}$

4. $\frac{7}{(x+1)}-\frac{4}{(x-2)}-\frac{3}{(x-2)^2}$

5. $\frac{4}{(x-7)}-\frac{5}{(x+3)}+\frac{6}{(x+3)^2}$

6. $\frac{2}{(x-5)}+\frac{4}{x}-\frac{8}{x^2}$