Outcome 1 – Distinct Linear Factors

Key Facts/Formulae:

When every factor is linear and distinct, each linear factor gets its own partial fraction.

E.g. $\frac{3x-22}{(x+1)(x-4)} = \frac{A}{(x+1)} + \frac{B}{(x-4)}$

Worked Example:

Express $\frac{3x-22}{x^2-3x-4}$ in partial fractions.

1. Factorise the denominator.

$x^2 - 3x - 4 = (x+1)(x-4)$

2. Begin process with the general formula

Let $\frac{3x-22}{(x+1)(x-4)} = \frac{A}{(x+1)} + \frac{B}{(x-4)}$

3. Multiply all through by the LCM of the denominators $(x+1)(x-4)$

$3x - 22 = A(x-4) + B(x+1)$

4. Sub in an 'x' value (preferably a root) $x = -1$

$-25 = -5A \qquad A = 5$

5. Sub in another 'x' value (preferably a root) $x = 4$

$-10 = 5B \qquad B = -2$

6. Answer the question!

$\frac{3x-22}{x^2-3x-4} = \frac{5}{(x+1)} - \frac{2}{(x-4)}$

Questions…

Express each of the following in partial fractions.

1. $\frac{4x-14}{(x+1)(x-5)}$

2. $\frac{8x+14}{(x-2)(x+3)}$

3. $\frac{12x-33}{x^2-5x+4}$

4. $\frac{3x+24}{x^2+9x+18}$

5. $\frac{4x-64}{x^2-5x-14}$

6. $\frac{-x-5}{x^2+6x+8}$

Answers

1 $\dfrac{3}{(x+1)}+\dfrac{1}{(x-5)}$

2 $\dfrac{6}{(x-2)}+\dfrac{2}{(x+3)}$

3 $\dfrac{7}{(x-1)}+\dfrac{5}{(x-4)}$

4 $\dfrac{5}{(x+3)}-\dfrac{2}{(x+6)}$

5 $\dfrac{8}{(x+2)}-\dfrac{4}{(x-7)}$

6 $\dfrac{1}{2(x+4)}-\dfrac{3}{2(x+2)}$